QUADRATIC PROJECTIONS FOR
SAXOPHONE QUARTET

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ABSTRACT

Quadratic Projections is a saxophone quartet in three movements using three projected pitch collections, each paired with a separate mathematical concept for organization, pitch groupings, tonal levels, and form. The instrumentation is soprano saxophone doubling on guiro and clarinet, alto saxophone doubling on vibra-slap and bass clarinet, tenor saxophone doubling on triangle and flute, and baritone saxophone doubling on rain stick and oboe.

Projection is a term used by Howard Hanson to describe the stacking of sonorities, intervals or chords, one above the other.† The trichord (three-note chord) used in Quadratic Projections, consisting of one perfect fifth, one major third and one minor third, can be expressed as either a major triad or a minor triad. The mathematical concepts used are pi carried to the 20th decimal place, the Fibonacci series expressed as the golden ratio, and Pascal’s triangle.

The first movement pairs a minor triad with a major triad projected at the tritone, using pi to the 20th place – $pi = 3.14159265358979323846$. Each digit of pi determines the 21-pitch grouping. The first four digits are used to determine the tonal levels of the movement.

Movement two superimposes a minor triad upon itself. The form of the second movement is based on the Fibonacci series expressed as the golden ratio, also called the divine proportion. The golden ratio is the organizing factor of the form of the movement. A texture and tempo

change occurs at the golden ratio and a rhythmic change occurs at the golden ratio of the golden ratio.

The third movement projects minor or major triads at the interval of a major second. The mathematical concept employed is Pascal’s triangle, an array of binomial coefficients beginning with “1” at its apex. As in Pascal’s triangle, the movement begins with a single instrument gradually expanding to the entire quartet. The form consists of a five-part palindrome followed by a short coda.
DEDICATION

*Quadratic Projections* is dedicated to my first composition teacher, Dr. W. Francis McBeth (1933-2012). This work would not have been possible without the solid theory and composition foundation provided by Dr. “Mac.” I am blessed to have called him my teacher, my mentor and my friend. I miss him greatly and am saddened that I can never again discuss compositions, music and life with him. The world has lost a truly wonderful man and a gifted composer.
ACKNOWLEDGMENTS

I am grateful to many individuals who have left their imprint on me throughout my doctoral program at the University of Alabama. I thank the members of my committee: Dr. Craig First and Dr. Marvin Johnson for helping me expand my compositional tools and skills, Dr. Thomas Robinson for his guidance in my writing, Dr. Diane Schultz for her encouragement and support, and Dr. Theodore Trost for graciously agreeing to serve on my committee. I thank Dr. Linda Cummins for her guidance, patience and understanding, and for her ability to keep me focused and on schedule.

A special thanks is given to my other composition teachers through the years: Dr. W. Francis McBeth, Dr. Newell K. Brown, Dr. Martin Mailman, Dr. Cindy McTee, and Dr. Dinos Constantinides. I am forever grateful to have studied with and learned from them.

Heartfelt appreciation is extended to the members of the Cahaba Saxophone Quartet, Kim Bain, Lori Ar dovino, Sallie White, and Jon Remley, for their inspiration for Quadratic Projections and for their eagerness to perform new music. The greatest debt is owed to my family for their unwavering support, especially my husband, Jon, and daughter, Lian, for their love, patience and faith in me while I worked on this degree. I could not have completed it without them by my side.
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INTRODUCTION

*Quadratic Projections* is a three-movement saxophone quartet. Pitch collections in each movement are derived from triadic projections as described by Howard Hanson. Pitch groupings, tonal levels, forms and texture densities are determined from three distinct mathematical concepts.

Instrumentation

The instrumentation of *Quadratic Projections* is soprano saxophone doubling on guiro and clarinet, alto saxophone doubling on vibra-slap and bass clarinet, tenor saxophone doubling on triangle and flute, and baritone saxophone doubling on rain stick and oboe. Percussion doublings occur in the first movement; the third movement employs woodwind doublings.

Explanation of Projection

In *Harmonic Materials of Modern Music: Resources of the Tempered Scale*, Howard Hanson defines “projection” as the stacking of sonorities, intervals or chords, one above the other. The simplest projection is that of perfect fifths — C, G, D, A, E, B, etc. Hanson used projections to build six-note “scales” he referred to as “hexads.” The first six tones of the perfect fifth projection reduced within an octave produces the hexad C – D – E – G – A – B. Hexads can be used both melodically and harmonically. When the projected interval produces fewer than six

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tones, such as the major third projection \((C - E - G\# - B\# [C])\), where each projected third will continue producing the same three pitches, Hanson chooses an interval, such as a perfect fifth, to start a new projection. \((C - E - G\#) + (G - B - D\#) = (C - D\# - E - G - G\# - B)\). Example 1 shows the perfect fifth and major third projections as hexads.

\[\text{Perfect 5th projection} \quad \text{Hexad} \quad \text{Major 3rd projection} \quad \text{Hexad}\]

**Example 1:** Simple projections of perfect fifth and major third.

**Pitch Collections**

The pitch collections for each movement of *Quadratic Projections* are derived from a projected \(pnm\) trichord \([0, 3, 7]\). Hanson’s analysis theory defines six classifications of intervals: \(p\), perfect fifth or perfect fourth; \(m\), major third or minor sixth; \(n\), minor third or major sixth; \(s\), major second or minor seventh; \(d\), minor second or major seventh; and \(t\), the tritone. Therefore, the \(pnm\) trichord consists of one perfect fifth, one major third and one minor third, which can be expressed as either a major triad or a minor triad.

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Mathematical Concepts

Each movement is based on a distinct mathematical concept. The three concepts are $pi$, the golden ratio, and Pascal’s triangle.

$Pi$ is the ratio of any circle's circumference to its diameter, approximately equal to 3.14 in decimal notation. Many formulae in mathematics, science, and engineering involve $pi$, making it one of the most important constants in mathematics.³

The golden ratio is the unequal division of a line such that the ratio of the smaller part to the larger is the same as that of the larger to the entire line, approximately 1:1.618, described in mathematics as the ‘division in extreme and ratio.’ In contemporary parlance, the term ‘golden number’ is used in the context of a natural phenomenon or a man-made object. It has often been used to produce harmonious proportions in, for example, architecture, fine art and sculpture, and there have been attempts to detect it in musical forms. Some 20th-century composers have used it intentionally.⁴ The golden ratio, illustrated in Example 2, is also called the golden mean, the golden section or the divine proportion.

\[
\begin{array}{c}
| & A & + & B & | \\
\hline
A & & B \\
\end{array}
\]

Example 2: Golden ratio - $A+B$ is to $A$ as $A$ is to $B$.

---


Pascal's triangle is an array of binomial coefficients named for the French mathematician Blaise Pascal, although other mathematicians in India, Persia, China, Germany, and Italy studied it centuries earlier.\(^5\) Pascal’s triangle begins with “1” at its apex and continues downward in a triangular pattern as diagramed in Example 3. Each successive number in the series is the sum of the two numbers immediately above.

\[
\begin{array}{cccc}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1 \\
\ldots
\end{array}
\]

**Example 3: Pascal's Triangle.**

**Movement 1 – Pi - 3.14159265358979323846**

The first movement pairs two major triads or two minor triads at the interval of a tritone, producing the pitch collections in Example 4, from which melodic and harmonic material is derived. Example 5 shows the movement’s pitch order.

**Example 4: Major or minor triads projected at the tritone.**

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Example 5: Pitch order for movement 1.

The first movement carries $\pi$ to the 20th place, where $\pi = 3.14159265358979323846$. The digits of $\pi$ are used to determine pitch groupings and tonal levels. Example 6 shows the groupings of the first six digits. The movement continues this pattern until 21 digits of $\pi$ are completed. The determination of tonal levels using the first four digits of $\pi$ is charted in Example 7.

Example 6: Pitch groupings based on the digits of $\pi$. 
<table>
<thead>
<tr>
<th>Digit of $\pi$</th>
<th>Tonal level</th>
<th>Measure number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Bb</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>C#</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>Bb</td>
<td>23</td>
</tr>
<tr>
<td>Return</td>
<td>C</td>
<td>29</td>
</tr>
</tbody>
</table>

Example 7: Tonal levels derived from the digits of $\pi$.

Movement 2 – The Divine Proportion

The second movement superimposes a minor triad upon itself. Example 8 illustrates the projection and resulting pitch collection, which is introduced harmonically, then developed melodically. Example 9 illustrates a melodic line derived from this pitch collection.

Example 8: Minor triad superimposed upon itself.
The form of the second movement is based on the Fibonacci series expressed as the golden ratio, also called the divine proportion. Example 10 illustrates the golden ratio as the organizing factor of the form. A rhythmic change occurs at letter B, the golden ratio of C.

Example 10: Form of movement 2 using the golden ratio.

Movement 3 – Pascal’s Triangle

The third movement projects major or minor triads at the interval of a major second. Example 11 illustrates two possible outcomes of this projection. The opening section, shown in Example 12, combines the use of major and minor triads. The non-triadic melody in the second and third sections, shown in Example 13, is derived from these pitch collections.
Example 11: Major or minor triads projected at the major 2nd.

Example 12: Melody outlining triads, measure 1.

Example 13: Non-triadic melody derived from pitch collection, measure 34.

As in Pascal’s triangle, Movement 3 begins with a single entity, the alto saxophone, and gradually expands to include the entire quartet. The form, illustrated in Example 14, is a five-part palindrome, as is the fifth row of the triangle. The movement concludes with a short coda.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>B</th>
<th>A</th>
<th>coda</th>
</tr>
</thead>
<tbody>
<tr>
<td>measures 1-29</td>
<td>measures 30-55</td>
<td>measures 56-87</td>
<td>measures 88-99</td>
<td>measures 100-124</td>
<td>measures 125-129</td>
</tr>
</tbody>
</table>

Example 14: Palindromic form of movement 3.
INSTRUMENTATION

B-flat Soprano Saxophone, Guiro, B-flat Clarinet

E-flat Alto Saxophone, Vibra-slap, B-flat Bass Clarinet, Triangle

B-flat Tenor Saxophone, Triangle, Flute

E-flat Baritone Saxophone, Rain Stick, Oboe

Transposed score
3
Pascal's Triangle

\( \frac{1}{= \text{100}} \)

Soprano Saxophone

Alto Saxophone

Tenor Saxophone

Baritone Saxophone

\[
\begin{align*}
&5 \\
&\text{Sop. Sax.} \\
&\text{Alto Sax.} \\
&\text{Ten. Sax.} \\
&\text{Bari. Sax.}
\end{align*}
\]

\[
\begin{align*}
&9 \\
&\text{Sop. Sax.} \\
&\text{Alto Sax.} \\
&\text{Ten. Sax.} \\
&\text{Bari. Sax.}
\end{align*}
\]