ENHANCEMENT OF EXTENDED SURFACE HEAT TRANSFER

USING FRACTAL-LIKE GEOMETRIES

by

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ABSTRACT

This work investigates a technique to improve extended surface heat transfer through the use of fractal-like geometric patterns. When fractal-like geometries are considered, significant gains in the available surface area for fins can be achieved without large increases in fin volume or mass. For certain fractal patterns, the surface area of a fin can even be increased while reducing the mass of the fin. This would provide direct benefit for situations where the extended surface volume is restricted or minimized weight is desired. Fractal-like geometries are presented to increase the effectiveness and effectiveness per unit mass of fins for natural convection heat transfer as well as increase effectiveness per mass for radiation heat transfer.

Common extended surface heat transfer methods and developments were reviewed to obtain an understanding of the focus and limitations of previous work. Based on this literature review, it has been observed that the use of fractal-like geometries has been utilized for engineering applications. However, the use of fractal-like geometries for extended surface heat transfer has not been studied and therefore justified an investigation into their behavior.

In an initial investigation, fractal-like fins were manufactured in the patterns of the baseline and first three iterations of the Sierpinski carpet (base width of 0.1016 m, 0.0508 m, 0.0254 m) and modified Koch snowflake (base width of 0.1016 m) in order to quantify practicality and thermal performance. It was observed that fractal-like fins could result in increased fin effectiveness per unit mass by as much as 59%. Motivated by the initial experimental results, subsequent studies utilized computational modeling with a commercially available, industry standard computational modeling program. A computational investigation of
natural convection heat transfer from fractal-like fins was able to further support conclusions of the experimental results as well as model an additional iteration. Fin effectiveness per unit mass was increased by a minimum of 37% for the conditions tested. Finally, a computational investigation of radiation heat transfer from fractal-like fins showed that fin effectiveness per unit mass increased by a minimum of 25% for the conditions tested.
LIST OF ABBREVIATIONS AND SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area, m²</td>
</tr>
<tr>
<td>c_p</td>
<td>specific heat</td>
</tr>
<tr>
<td>c</td>
<td>constant</td>
</tr>
<tr>
<td>d</td>
<td>constant</td>
</tr>
<tr>
<td>F</td>
<td>view factor</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient, W/m²·K</td>
</tr>
<tr>
<td>i</td>
<td>coordinate direction</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity, W/m·K</td>
</tr>
<tr>
<td>m</td>
<td>mass, kg</td>
</tr>
<tr>
<td>N</td>
<td>Stark number, $\varepsilon_r \sigma w T_b^3/k$</td>
</tr>
<tr>
<td>n</td>
<td>iteration index</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number, hw/k</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>Q</td>
<td>heat rate, W</td>
</tr>
<tr>
<td>$q''$</td>
<td>heat flux, W/m²</td>
</tr>
<tr>
<td>R</td>
<td>rim surface area ratio, 1-A_i(n)/A_e(n)</td>
</tr>
<tr>
<td>S</td>
<td>surface</td>
</tr>
<tr>
<td>t</td>
<td>fin thickness, m</td>
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</tbody>
</table>
T      temperature, K
w      fin base length, m
x_n    normal direction
Z      ratio of fin base temperature to fin tip temperature
α      absorptivity
ΔT     average fin to ambient temperature difference, K
Γ      width/thickness ratio
ε      fin effectiveness
ε_r    emissivity
η      fin efficiency
θ      dimensionless temperature, (T_b - T_∞) / (T_1 - T_∞)
Λ      surface area ratio, A_s(n) / A_s(0)
ρ      fin density, kg/m^3
ρ_T    reflectivity
σ      Stefan-Boltzmann constant, W/m^2-K^4
Φ      face area ratio, A_f (n) / A_f (0)

Subscripts
avg    average
b      base
c      corrected
f      face
I      directional index
L      loss
r radiation, rim
rad radiating surface
s surface
t tip
$T, i$ incident
$\infty$ ambient

Materials
Al aluminum
Cu copper
Fe iron
Ti titanium
ACKNOWLEDGEMENTS

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CHAPTER 1
INTRODUCTION

Background

Fractals, as first mentioned by Mandelbrot [1.1], are a group of shapes and surfaces that cannot be described using classical geometric classifications and are very efficient at filling space. Fractals produce irregular shapes and surfaces by repeating geometric patterns at infinitely smaller scales with characteristics that include self-similarity and non-integer dimension. Fractals have been used in many areas related to engineering applications. Baliarda et al. [1.2] and Vinoy et al. [1.3] have demonstrated the use of fractal patterned antennae to increase performance while utilizing less volume. Coppens [1.4] showed that the self-similarity of fractal patterns could be used to create an effective distributor system for chemical reactions. Samavait et al. [1.5] developed a parallel plate capacitor using fractal geometries and improved capacitance per area by up to 130%. Mitra et al. [1.6] developed a technique using fractals to compress images which utilizes less memory than standard techniques while maintaining image quality. Onda et al. [1.7] showed how a hydrophobic surface with a surface fractal pattern could create surfaces that are more water repellant than those with chemical surface modifications.

The ability of fractals to increase heat transfer has been observed by Lee et al. [1.8]. In order to combat temperature induced error from overheating in a laser interferometer, a third order fractal Hilbert Curve was used to increase the heat transfer area in the instrument without increasing its size. By increasing the heat transfer area, a significant reduction in measurement error was achieved. Van Der Vyver [1.9] investigated a a tube-in-tube heat exchanger based on
using the quadratic Koch island fractal pattern for the inside tube using computational modeling software. The model indicated that with each fractal iteration, an increase in heat transfer by a factor of two was achieved. Meyer and Van Der Vyver [1.10] continued the work of Van Der Vyver [1.9] through both an analytical solution and experimental results. The analytic solution was found to be in agreement with the computational investigation while the experimental results of a tube-in-tube heat exchanger prototype resulted in an increase in heat transfer of a factor of 2.1 and 3.9 for the first and second iteration respectively.

Another research interest involving fractals in engineering has been the effects of fractal surface geometries and boundaries. The conductivity of an object has been able to be structured through the arrangement of fractal patterns. Kacimov and Obnosov observed [1.11] that the overall thermal conductivity could be adjusted to meet needs by arranging separate materials with unique thermal conductivities using a Sierpinski carpet pattern. Adrover [1.12] noted that the thermal boundary layer for convection is greatly affected by the surface fractal dimension such that surfaces with greater fractal dimension result in greater heat transfer. It was noted that the total heat dissipation across the boundary layer is proportionate to the surface fractal dimension. Blyth and Pozrikidis [1.13] examined fractal surface effects on heat transfer by utilizing fractal surface patterns that resulted in protrusions and examining the displacement length as a measure of heat transfer. It was observed that after five to ten iterations, convergence of the displacement length was achieved. Therefore, while infinite fractal iterations result in infinite increases in surface area, heat transfer is only increased for a limited number of iterations. A similar deduction was made when Brady and Pozrikidis [1.14] also concluded that the diffusion rates from a fractal surface converged after four to five iterations for the fractal patterns tested.
Branched geometries, which are similar in many ways to fractal geometries, can also be examined and can give insight into fractal behavior. In a two part study, Plawsky [1.15,1.16] was able to demonstrate that the fin effectiveness of a branching geometry under natural convection conditions increases with subsequent branch generations. However, after a finite number of branch generations, the increase in fin effectiveness was negligible such that heat transfer increases are restricted. Lee and Lin [1.17] further expanded the investigation of Plawsky [1.15] by examining branching networks for forced convection. As fluid velocity is increased, the optimum number of branch levels to maximize heat transfer is reduced such that the number of branch levels required for forced convection is less than the number of branch levels for natural convection. Xu et al. [1.18] investigated the optimal branching ratio as a function of minimum resistance for heat conduction and found it to be 0.707, as opposed to the optimal flow branching ratio of 0.7937 from Murray’s Law. Hong et al. [1.19] compared a parallel channel heat sink with a fractal-shaped branching flow network and found that the fractal network resulted in a more uniform temperature distribution while reducing thermal resistance for the same pumping power. Wang et al. [1.20] noted that the reliability of fractal-shaped branching network is higher than parallel channel networks as they are less susceptible to flow blockages, due to alternative flow pathways. Despite the ability of fractal-like branching to increase heat transfer, Gao et al. [1.21] showed that fractal branching can be used to create more effective insulators. Using goose down fibers arranged in branching networks, thermal resistance was increased measurably by assembling branching networks of low thermal conductivity materials with large numbers of branch levels.

Another study that can lend insight into the behavior of certain fractal patterns, particularly the Sierpinski carpet which is utilized in this dissertation, are perforations in fins. In
a series of studies, AlEssa et al. [1.22-1.24] examined the performance of plate fins with varying geometric perforations. First, with square perforations considered, heat transfer per unit mass could be increased over that of a standard plate fin for natural convection. When rectangular perforations and triangular perforations were considered in subsequent studies, heat transfer per unit mass was found to increase over that of a standard plate fins as well. A notable finding of the studies was that the optimum orientation, i.e. vertical vs. horizontal, varied based on the thermal conductivity and fin thickness. Sahin and Demir [1.25-1.26] examined the effects of perforated fins on the thermal performance of a heat exchanger. It was found that larger Nusselt numbers, corresponding to an increase in heat transfer, could be achieved from the use of circular and square perforated pin fins. Despite increased pressure drop as a result of the perforations, the heat transfer per pumping power was found to increase on average by a factor of 2.

Sparrow and Ortiz [1.27] investigated a plate with circular perforations with fluid flow normal to the plate. An experimental correlation of the Nusselt number was developed based on the Reynolds and Prandtl numbers gathered from measured flow properties. Similarly, Kutscher [1.28] developed an experimental correlation of the Nusselt number for a plate with circular perforations with fluid flow tangential to the plate. It was shown by Akyol and Bilen [1.29] that the thermal performance of an array of plate fins could be increased by perforations creating hollow channels down the width of the fin. Although aligned arrays resulted in heat transfer per pumping power greater than that of solid fins, the staggered arrays were not able to overcome the increase in pumping power required. Sara et al. [1.30] examined similar conditions for an array of plate fins with perforations through the fin thickness. Unlike the study performed by Akyol and Bilen [1.29], Sara et al. [1.30], varied the inclination angle and diameter of the perforations
to find that increases in heat transfer per pumping power of up to twenty percent could be achieved. Elshafei [1.31] examined perforations for fins with hollow shells. The author noted that a combination of hollow fins with perforations can result in increases in heat transfer greater than that of a solid fin of the same diameter.

Comparably, Prasad and Gupta [1.32] removed a semicircular section of a rectangular fin at the tip in an attempt to increase heat transfer per unit mass. For distinct radii, the semicircular cuts were found to increase the fin effectiveness and fin efficiency. The ability of perforations to increase the thermal performance of fins subject to turbulent flow conditions was investigated by Shaeri et al. [1.33-1.35]. Perforations were inserted into an array of fins for both turbulent cross flow and parallel flow. For fins subject to cross flow, small increases in heat transfer were achieved with considerable decreases in pumping power. For fins subject to parallel flow, heat transfer was significantly increased with a corresponding considerable decrease in pumping power. The reduced pumping power was found to increase as a result of reduced turbulent wake due to the added perforations. For both cases, the fin effectiveness per unit mass was increased with many cases also resulting in increased fin effectiveness. An experimental correlation for the Nusselt number for a perforated plate inserted normal to the flow was developed by Dorignac et al. [1.36]. The correlation was based on pitch, hole diameter, plate surface area, and flow velocity with the most significant contribution being the pitch. Gorla and Bakier [1.37] increased fin heat transfer when compared to a solid fin of the same dimensions by utilizing porous fins.

Thermal performance of extended surfaces has been subject to numerous investigations for natural convection. Plate-fin heat sinks were evaluated by Yüncü and Anbar [1.38] who found that for a given heat rate, optimal fin spacing decreases as fin height increases. Razelos and Georgiou [1.39] developed a guideline for longitudinal, annular, and pin fins. A fin
effectiveness value of 10 was found to be the minimum value that justified the addition of fins. Bar-Cohen et al. [1.40] developed optimal fin spacing and fin aspect ratios for a plate-fin heat sink for both constant mass and constant volume. Despite the standard analysis of heat sinks as having an isothermal base temperature, Van De Pol and Tierney [1.41] developed a correlation of the Nusselt number for varying base temperature. Mahmoud et al. [1.42] compared optimal fin dimensions for micro-scale heat sinks and macro-scale heat sinks to assess the validity of using the same model for both scales. It was concluded that micro-scale heat sink behavior cannot be accurately predicted using macro-scale models and analysis. Fujii and Imura [1.43] investigated the fin inclination angle and its effect on the performance in a heat sink. A Nusselt number correlation was developed and it was shown that boundary layer separation increased with higher inclination angle. Sunden [1.44] found that the buoyant natural convection forces concentrate at the base of the fin as the ratio of convection to conduction increases.

The shape of a fin has also proved to be critical to the heat transfer performance. Sikka et al. [1.45] investigated the comparative performance of heat sinks in which longitudinal fins were replaced by fluted and wavy fins. Fluted fin heat sinks were found to increase heat transfer by up to 9% while wavy fins were found to increase heat transfer by up to 6%. Lin and Jang [1.46] investigated the effects of an elliptical annular fin in place of a circular fin. When an ellipse was compared to a circle with the same perimeter, the elliptical efficiency was higher and increased with increasing elliptical aspect ratio. Another investigation in the comparison of elliptical and circular fins was performed by Li et al. [1.47]. This study however examined the geometric effect on the performance of a pin fin. Elliptical pin fins were able to result in higher heat transfer coefficients while reducing the pressure drop across the array of fins. Razelos and Satyaprakash [1.48] investigated a trapezoidal shaped pin fin for optimization based on heat rate
or volume and found an optimal shape aspect number of 1.5. Kang [1.49] investigated adding a rectangular base to a triangular profile fin and found that for a minimized volume optimization, the simple triangular profile achieves greater heat transfer than the modified design.

Hexagonal pin fins were investigated by Yakut et al. [1.50] and the optimal dimensions were found, with fin width greatly affecting the pressure drop across the array of fins. Sparrow and Grannis [1.51] investigated the pressure drop across an array of diamond shaped pin fins to find that the pressure drop decreased as the diamond shape became more elongated. A correlation of the friction factor was also developed to account for the pressure drop for given geometry and flow conditions. Tanda [1.52] found that the heat transfer coefficient for diamond shaped pin fins varied proportional to the pitch, with increases in heat transfer of up 340% beyond that of a flat pin fin. Al-Arabi and El-Reidy [1.53] compared the heat transfer from flat rectangular and circular plates to investigate corner effects. The average heat transfer coefficient for a circular plate was found to be approximately that of a square plate with base equal to the diameter of the circle. Corners were found to have little influence on the overall behavior of the heat dissipation.

A critical factor in heat transfer is the interaction between the combined conjugate effects of natural convection and radiation. An important role in radiation heat transfer is its influence on fin performance. It can be seen in published literature that thermal radiation accounts for a significant portion of heat transfer in heat sinks operating under what is typically considered natural convection only. Sasikumar and Balaji [1.54] studied the contributions of both radiation and natural convection in a plate fin heat sink and found that radiation could contribute up to 55% of the heat transfer for heat sinks with short fins. It was noted that the contributions of radiation was reduced with increasing fin height. Rao and Venkateshan [1.55] also showed that
the interaction between radiation and natural convection must be accounted for as radiation can account for 25-40% of the heat dissipation in the array. In a study of a strip fin heat sink, Guglielmini et al. [1.56] noted that radiation could contribute up to 40% of the heat dissipation. It was also shown that a staggered fin arrangement resulted in increased heat transfer for a vertical arrangement. For pin fins, with the mutual interaction of radiation between fins, Gerencser and Razani [1.57] found that the optimal shape resembled a parabolic spine, with a triangular cone providing comparable effectiveness. Rao et al. [1.58] developed a system for modeling conjugate heat transfer from a heat sink by modeling it as a two fin enclosure, with each end having half the fin thickness. The ends of the heat sink were modeled separately, such that only two sets of channels have to be modeled for analysis of an entire heat sink.

Rea and West [1.59] highlighted the contribution of thermal radiation heat transfer to natural convection by finding that approximately 25% of the total heat transfer in a plate fin heat sink is due to radiation. Khor et al. [1.60] similarly found that thermal radiation contributed up to 30% of the total heat sink heat transfer. In their investigation, they analyzed the effects of ignoring and including radiation in natural convection analysis. It was advised that if radiation was included, view factors must be included as their exclusion could result in errors of the natural convection model by up to 60%. With a similar assumption of thermal radiation accounting for a significant portion of the heat transfer in a natural convection heat sink, Abramzon [1.61] developed a general closed form solution for the shape factor. Similarly, it was stated that thermal radiation accounted for over 20% of the total heat transfer. Yu et al. [1.62] noted that heat transfer increases of up to 12.3% could be accomplished if thermal radiation was accounted for in the heat sink design. A more compact design could also be utilized by accounting for thermal radiation. Another closed form solution of the view factor was developed
by Shabany [1.63]. A maximum error of 11% was found for the view factor corresponding to fin spacing approaching infinity.

Kobus and Oshio [1.64] further studied the combined effects of mixed convection and radiation heat transfer. It was demonstrated that a radiation heat transfer coefficient could be defined such that the overall heat transfer coefficient could be found by adding the convective heat transfer coefficient to the radiant heat transfer coefficient. Chen and Fang [1.65] developed a simple computational model to evaluate the performance of a fin such that only the base temperature and tip temperature were estimated. This allowed for the performance characteristics to be found with only three to four iterations and negligible effects when compared to benchmark solutions. Karabacak [1.66] found that for a circular fin around an annular heat pipe, the contribution of radiation heat transfer decreases as the ratio of the fin spacing to fin diameter decreases. The interactions between radiation and convection were further studied by Nouanegue et al. [1.67]. In an open cavity, the surface emissivity was found to greatly alter the velocity and temperature distribution. It was also noted that increased emissivity resulted in higher Nusselt number. Rabhi et al. [1.68] investigated the effects of adding cavities between an isothermal hot and cold wall. It was shown that as the number of partitions increased, the total heat transfer decreased and noted that the partitions acted like radiation shields. Likewise, Antar [1.69] found that increasing the number of cavities between two walls could decrease the total heat transfer, with a thermal resistance increase of up to 43% for the considered cases.

The effects of the surface geometry have been shown to affect the heat transfer from extended surfaces greatly. Square ribs along the surface of a fin have been found to increase the thermal performance by up to 30% in an investigation by Firth and Meyer [1.70]. Similarly, Kukulka and Fuller [1.71] noted increases in thermal performances of up to 38% for diamond
shaped surface protrusions and better flow distribution along the surface. Wang et al. [1.72] found that grooved surfaces could lead to increased heat transfer of up to 25% and moderate gains in pressure loss due to its ability to affect flow by creating localized suction and blowing at the groove/surface interface. Surface dimples have also been seen to have effects similar to grooves. Rao et al. [1.73] found that dimples placed on the baseplate of a pin fin heat sink could increase heat transfer by up to 8% while reducing the friction factor by up to 18%. Chang et al. [1.74,1.75] found that surface dimples in both concave and convex configuration result in higher heat transfer per pumping power in a channel flow, with convex dimples having the larger heat transfer per pumping power.

Another problem studied in heat transfer is the attempt to maximize the performance of a heat sink by modifying existing conditions. Yu et al. [1.76] developed a plate fin heat sink that placed pin fins in between the longitudinal fins for increased heat transfer per pumping power of up to 20%. This method was presented as a technique for modifications of existing plate fin heat sinks. Sparrow et al. [1.77] investigated the enhancement of cooling in electronic equipment by placing barriers to alter the flow before reaching the desired cooling device. While this was found to greatly increase the heat transfer, it required additional fan power. In a comparison study, Jonsson and Moshfegh [1.78] evaluated the thermal performance and pressure drop of strip fin heat sinks versus pin fin heat sinks. While the thermal resistances of strip fin heat sinks were comparable to pin fin heat sinks, the pressure drop across pin fin arrays were larger than strip fins. Yeh [1.79] developed a formulation for the optimal dimensions of a pin fin heat sink based on variable heat transfer coefficient such that it could be extended from natural convection to forced convection.
Orientation of a heat sink is also an important factor in the heat sink performance. For the pin fin heat sink investigated by Sparrow and Vemuri [1.80], it was found that for small amounts of pins, vertical and horizontal orientations were comparable. However, as number of fins increased, the vertical orientation became increasingly more effective. A similar behavior was noticed for square fins by Huang et al. [1.81] with the exception of short fins. The investigation found that for short fins, which had finning factors less than 2.7, the horizontal arrangement had higher heat transfer performance than the vertical arrangement. It was also shown that for a simple base plate, the horizontal arrangement resulted in greater heat transfer for all conditions. Another external factor that could affect heat transfer is the humidity of air. Kundu [1.82] found that the heat transfer from fins was significantly increased when the fins were under the fully wet condition, with little dependence on the relative humidity.

Constructal fins are patterns similar to fractals in that they are also inspired by nature and have been able to increase heat transfer. Using constructal fins, Bejan and Almogbel [1.83] were able to minimize thermal resistance using a T shaped branching network for a point-to-volume heat source. Lorenzini et al. [1.84] also minimized thermal resistance of a point-to-volume heat source by utilizing a T-Y branching network. By inserting a cavity between the initial T shaped branch, fin effectiveness was increased by over 32% compared to the previous T shaped branching network. Zimparov et al. [1.85] modified parallel and counter flow heat exchangers using constructal flow paths. Optimal branching angles and branching ratios were found to be dependent on the pumping power available.

The importance of radiation heat transfer can be easily seen in space related applications as it is the only means of rejecting waste heat to the surroundings. Krikkis and Razelos [1.86] optimized rectangular and triangular plate fins for radiation temperature subject to opening
angles between fins less than 180°. The fin height and fin thickness required to optimize heat transfer were found to be a function of the fin surface emissivity. The optimal ratio of the dimensionless radiation to conduction parameter was found to be 0.8464. Kumar et al. [1.87] performed an optimization based on weight to use the optimal number of fins for a plate fin heat sink. As the number of fins was increased, heat transfer increased greatly with low number of fins. A sharp initial increase in heat transfer was found until the optimal number of fins was reached with little increases in heat transfer for a number of fins greater than the optimal amount. A number of radiation heat transfer studies were conducted by Sparrow et al. [1.88-1.90] involving mutual irradiation between extended surfaces. It was noted that in order to maximize the thermal performance of a heat sink radiating to surroundings, the optimization of the heat sink must include effects of irradiation between both fins and the heat sink base. The heat transfer from fins was determined not to be linearly proportional to the emissivity of the fins. Fin efficiency was found to be adversely impacted by decreasing thermal conductivity. A final investigation involving isothermal fins around an annular heat pipe found that fin effectiveness was primarily a function of thermal conductivity, geometric parameters, and fin surface emissivity.

Krishnaprakas [1.91,1.92] found that the use of a plate fin heat sink for radiation heat transfer was limited as the optimal design parameters called for an array of short, slender fins. For plate fins arranged around an annular heat pipe, optimal parameters were found to be high emissivity fins with minimal opening angles, although the opening angle was restricted to a minimum of 40°. Ellison [1.93] used a circuit method resistance procedure to analyze the gray body shape factor for a rectangular plate fin array. In comparisons, it was noted that previous studies over or under predicted shape factors which could greatly affect heat transfer analysis.
Razelos and Krikkis [1.94] found an optimization for rectangular fins for a given volume and noted that the surface Biot number was the largest contributor. Another key determination was that the interaction between the base and fins for a tubular radiator was insignificant and could be ignored during thermal analysis. In a similar study with the use of trapezoidal fins instead of rectangular fins, Karlekar and Chao [1.95] discovered that decreasing fin emissivity resulted in an increase in the optimum number of fins. Kumar and Venkateshan [1.96] investigated a variable temperature annular fin array where the temperature gradient varied based on the heat transfer achieved by the array. For variable temperature arrays, it was noted that the fin effectiveness was reduced as fin emissivity was increased beyond the optimal value. Arlsanturk [1.97] examined a rectangular plate fin with temperature dependent thermal conductivity. The optimal plate dimension was found to vary by a value of up to 28% of a plate fin with constant thermal conductivity with the same optimized geometry.

Keller and Holdredge [1.98] developed a correlation for the fin efficiency of both rectangular annular and plate fins with tapered edges based on the taper angle. Naumann [1.99] compared the performance of tapered fins to rectangular straight fins. By tapering fins, it was found that the heat transfer per unit weight could be increased by up to 14%. It was also noted that the radiation heat transfer is proportional to 1/3 of the ratio of thermal conductivity to density. Schnurr et al. [1.100] found that all angles of taper increased the performance with the maximum value of heat transfer per unit mass achieved when the angle of taper resulted in a triangular tip of the fin. Chung and Zhang [1.101, 1.102] developed a design for a minimum mass longitudinal fin. The optimal shape with regards to heat transfer per unit mass was found to have a parabolic surface. An alternative to the parabolic shape so that ease of manufacturing could be incorporated resulted in a rectangular tip with insignificant impact on heat transfer. In
another study, Krishnaprakas and Narayana [1.103] found that a fin profile with a spine shape optimized heat transfer per unit mass. The optimized fin shape was found to be independent of mutual irradiation between fin elements with optimal fin length linearly proportional to heat flux. Wilkins [1.104] found that an optimally shaped parabolic face rectangular fin could result in a 39% reduction in mass over a rectangular fin with the same heat dissipation. Similarly it was found that a parabolic face pin fin could result in a 20% reduction in mass over a straight pin fin with the same heat dissipation. Murali and Katte [1.105] found that grooves and threads in pin fins could be used to increase radiation heat transfer on a mass basis. Grooves were found to increase effectiveness per mass up to 40% while threads were found to increase effectiveness per mass up to 20%.

The work presented here calls for the use of extended surfaces with fractal-like geometries. Based on the extensive literature survey conducted, there has been no previous investigation into the performance of fractal-like fins. However, the knowledge gained from the literature review can be used to assist with the understanding of the behavior of fractal-like fins.

Motivation

The use of extended surfaces is a thermal management technique used to augment heat transfer by increasing the available surface area and therefore the total heat dissipation. Extended surfaces can be found in most electronics, engines, industrial equipment, and a variety of other mechanical devices. The ability of an object to reject excess heat is a required task to ensure operability, and if not accomplished sufficiently, can result in device malfunction or even failure. Natural convection heat sinks are often utilized for heat dissipation because of their ability to reject heat passively. Passive enhancement is typically employed because of its reliability and capacity to operate without fans, which require additional power input. In most situations it is
ideal to utilize the least amount of space while still obtaining a high level of performance. This can often be challenging because the heat transfer from an object to its surroundings is proportional to the available surface area. Engineers such as Saini and Webb [1.106] have claimed that the performance of plate fins for air cooling has reached its limits of effectiveness. However, when fractal geometries are considered, significant gains in available surface areas can be achieved without large increases in fin size or mass. For certain fractal geometries, surface area can even be increased while reducing the mass of the fin. Fractal geometries have the potential to increase natural convection heat transfer for commercial and industrial applications, where portable electronics and industrial equipment could directly benefit from an increase in surface area and reduction of mass.

The dissipation of waste heat is also a critical function in space related thermal management. The waste heat generated onboard a satellite or any other orbiting equipment must either be converted into useful energy or rejected to its surroundings. This can be difficult however, as the only mode of heat transfer available for dispelling this excess heat is thermal radiation. A common technique for heat transfer from equipment in space is to utilize extended surfaces aligned such that incident solar radiation and reflected radiation from other surfaces is minimized [1.107]. Since thermal radiation heat transfer is also a function of the available surface area, increased heat transfer can be difficult to achieve without the use of higher volume/mass extended surfaces. A reduction in mass of heat transfer devices is beneficial as the cost associated with carrying a payload into space is typically around $10,000-$14,000 per kilogram [1.108].

In a wide-ranging literature survey, typical extended surface heat transfer advancements and optimum formulations were studied to find the limits of previous work. It was observed that
the use of extended surfaces with geometric fractal-like patterns had not been previously studied. Therefore, an investigation into the thermal performance and behavior of fractal-like fins was undertaken.

Objectives

The overall objective of the presented work is to quantify the heat transfer performance characteristics of extended surfaces using fractal-like geometries for both natural convection and radiation heat transfer. A comparison is developed between the fractal-like geometries and classical geometries found for common extended surfaces. In order to complete the objective for this work, three major tasks were undertaken as listed below:

1) An experimental investigation into the performance of fractal-like fins for natural convection heat transfer was conducted. Fractal-like fins were manufactured for the baseline case and first three fractal iterations utilizing the Sierpinski carpet and modified Koch snowflake patterns. The investigation was conducted for a range of three scales and performance was evaluated using fin effectiveness, fin effectiveness per mass, and fin efficiency.

2) A computational model, which was verified by experimental investigation and analytical solutions, was used to further evaluate the performance of fins with fractal-like geometry for natural convection heat transfer. The fins were modeled for the baseline case and first four iterations utilizing the Sierpinski carpet and modified Koch snowflake patterns. A fourth iteration allowed for a better understanding of how the performance characteristics behave with iterations. Computational modeling also allowed for a parametric study in which a larger range of parameters were evaluated to better understand the critical variables. Performance was evaluated using fin effectiveness, fin effectiveness per mass, and fin efficiency.
3) A computational model, which was verified by analytical solutions, was used to evaluate the performance of fins with fractal-like geometry for radiation to a free space environment. The fins were modeled for the baseline case and first four iterations utilizing the Sierpinski carpet and modified Koch snowflake patterns. Similar to the second task, a parametric study was conducted for a better understanding of the critical variables. Performance was evaluated using fin effectiveness, fin effectiveness per mass, and fin efficiency.

References


Bibliography


CHAPTER 2

NATURAL CONVECTION FIN PERFORMANCE USING FRACTAL-LIKE GEOMETRIES

Abstract

Results of a study into the use of fractal geometries for extended surface heat transfer enhancement are presented. Fractal geometries are shown to increase surface area significantly with patterns such as the Sierpinski carpet providing the potential for mass reduction. Two fractal geometries were used in this study, the modified Koch snowflake and the Sierpinski carpet. This study examines fin performance for the baseline cases (a triangular fin and a square fin) relative to the first three fractal iterations for both geometries. Constant heat rate conditions were applied to the base of the fins and the temperature distribution across the fins was observed using an infrared camera. Fin effectiveness and fin efficiency were calculated for each fin geometry in order to quantify the effects of using fractal geometries to improve fin performance. Based upon the observed results, fractal geometries can be used to improve fin performance, particularly when the decrease in fin mass is a performance criteria. As fins are used for passive thermal management in many industrial and electronic devices, the use of fractal-like geometries has wide reaching potential.

Introduction

The dissipation of waste heat is a critical function in many areas, ranging from electronics, engines, industrial equipment, and a variety of other mechanical devices. If the removal of waste heat is not performed sufficiently, it can lead to a device malfunction or

decreased performance. Typically, it is desired to utilize the least volume and mass of material while maximizing heat transfer. Since convective heat transfer is directly proportional to the available surface area, this can be a challenging task. In order to provide additional surface area for heat transfer extended surfaces are often employed. Fractal geometries are proposed to replace standard fin geometries as fractal geometries can lead to large increases in available surface area without large increases in fin size or mass. Furthermore, certain fractal geometries can result in increased surface area with a corresponding reduction of fin mass. These increases in surface area have the potential to increase natural convection for commercial and industrial applications where increased heat transfer performance or reduction of weight is essential. Another benefit of using fractal-like fins would be minimized cost for spacecraft thermal management due to a reduction in mass.

Fractals and heat transfer are not new topics of research interest. Lee et al. [2.1] observed that a major problem in precision instruments is temperature induced error caused by overheating, specifically in a laser interferometer used for experimentation. In order to combat the error, a third order fractal Hilbert Curve was used to increase the heat transfer area in the instrument without increasing its size. It was noted that by modifying Lee’s interferometer with the Hilbert Curve, results were obtained comparable to the most precise interferometer available on the market. Van Der Vyver [2.2] developed a computational model for a fractal based tube-in-tube heat exchanger utilizing a quadratic Koch island for the inside tube. It was shown that for each iteration of the fractal pattern, heat transfer increased by a factor of two. Meyer and Van Der Vyver [2.3] continued the work of Van Der Vyver [2.2] and explored an analytical model. The analytic model was found to be in correspondence to the computational model in that the heat transfer increased by a factor of two for each iteration. Furthermore, an experimental model
of the heat exchanger showed an increase of heat transfer by a factor of 2.1 for the first iteration and 3.9 for the second iteration. When pumping power was considered however; the heat transfer gains from the second iteration onward were overshadowed by the increase in pumping power required to achieve the same flow rate.

Fractal surface geometries and boundaries have also been a subject of research interest. Fractals have been shown as a method of controlling the conductivity of an object. It was shown through the work of Kacimov and Obnosov [2.4] that using Sierpinski patterns made of materials with unique thermal conductivities one can control the overall thermal conductivity of a composite material. As shown by Adrover [2.5], the fractal dimension of a surface has a substantial effect on the thermal boundary layer. It was noted that the overall heat rate across a surface is proportional to the fractal dimension of the surface, showing it to be beneficial to have surfaces with large fractal dimensions for heat transfer. This study was for micro-scale surfaces and does not necessarily apply to macro-scale behavior. Blyth and Pozrikidis [2.6] also studied the effects of fractal surfaces, but chose fractal patterns that resulted in surface protrusions. Using the displacement length, the author was able to characterize the surface effects on forced convection. It was shown that the displacement length converged toward a specific value between five and ten iterations, indicating that increases in heat transfer peak after a finite number of iterations. This conclusion was reinforced in a study by Brady and Pozrikidis [2.7], which showed that diffusion rates across a surface approached a limit after a finite number of iterations, typically around four to five iterations dependent on the fractal pattern.

Similarly to fractal geometries, branched geometries have also been studied and can give some insights into the behavior of fractals. In two studies by Plawsky [2.8, 2.9] it was shown that for natural convection using a branching geometry, the effectiveness increased with each branch
generation until peaking at a finite branch level, a result similar to the previously mentioned fractal investigations. However, efficiency of the branching was reduced with each subsequent branch generation. It was also noted that heat transfer rate was maximized when the branching occurred as close to the base as possible. Contracting and expanding branch networks were studied, with contracting networks achieving greater thermal performance. A continuation of the work of Plawsky [2.8] was performed by Lee and Lin [2.10]. The branching networks were no longer studied for natural convection, but for forced convection. It was noted that as Reynolds number is increased, less branching is required for optimization of heat transfer when compared to natural convection. The use of fractals in heat transfer is not limited to maximizing heat transfer though. Through the work of Gao et al. [2.11], it can be seen that fractal branching can be used to create excellent insulators. Fibers in goose down were investigated and it was found that thermal resistance can be increased drastically when low thermal conductivity materials are arranged with large branch levels.

Perforations can be found in many studies about fins and have parallels to the behavior of fins from patterns such as the Sierpinski gasket or Sierpinski carpet. Al-Essa et al. [2.12-2.14] conducted numerous studies on the effects of perforated fins and their related performance on plate fins. For a plate fin with square perforations, it was shown that heat transfer per weight could be increased with optimal square width and spacing for natural convection. This was also found to be true for rectangular perforations with locked aspect ratio as well as triangular perforations. The optimal orientation of the fins compared to the direction of gravity was also found to vary with regard to the fin thickness and thermal conductivity of the material. Similarly, Prasad and Gupta [2.15] attempted to increase heat transfer per weight through the removal of a semicircular piece at the tip of a rectangular fin. It was found that the effectiveness increased for
particular radii cuts and that the fin efficiency could also be improved. The performance of perforated fins in a heat exchanger was studied by Sahin and Demir [2.16, 2.17]. It was found that perforations on pin fins and square fins induced higher Nusselt numbers and an increase in heat transfer. Although pressure drop increased due to the perforations, heat transfer enhancement efficiency was found to increase on an order of 1.4-2.6 depending on the clearance ratio of the fins. The author also noted that the optimal clearance ratio was found to be unity. Murali and Katte [2.18] found that by removing pieces of a pin fin to imitate grooves, heat transfer per weight was increased by approximately 20%.

Sparrow and Ortiz [2.19] examined the effects of circular perforations on a plate exposed to an oncoming flow normal to the plate. By measuring the flow properties, the author was able to develop a correlation of the Nusselt number based on the Prandtl number and Reynolds number in the perforation. Hollow rectangular fins have also been applied normal to an oncoming flow. In a closely related work, Kutscher [2.20] developed a correlation of the Nusselt number similar to that of Sparrow and Ortiz [2.19] for a perforated plate with a crosswind tangential to the plate. The correlation was also able to take into account suction velocities normal to the plate. Akyol and Bilen [2.21] showed than an aligned array of fins with hollow channels running down the width of a fin can result in heat transfer enhancement capable of overcoming the increased pumping power. While staggered arrays also resulted in a larger increase in heat transfer, the pumping power required did not offset the gain. A similar work was performed by Sara et al., [2.22] with the perforations running through the thickness of the fin. The study was more in depth as the author varied the inclination angle of the holes as well as the diameter. It was found that energy gains of up to twenty percent could be achieved through perforations. Perforations in hollow fins have been shown to increase heat transfer by Elshafei
The author noted that a combination of hollow fins with perforations can result in heat transfer improvements over a solid fin of the same diameter. An investigation into the effects of perforations in turbulent flow was performed by Shaeri et al. [2.24, 2.25]. Perforations were inserted into fins tangential to the flow. This resulted in significant decreases in the pumping power despite small increases in heat transfer. However, when perforations were inserted into fins normal to the flow, heat transfer was increased as well as pumping power reduced. It was also noted that as the number of perforations was increased, the turbulent wake behind the fins was reduced.

Other investigations into fin performance have included asymmetric heat transfer and transient effects. Kang and Look [2.26-2.27] investigated the effect of asymmetry on the performance of extended surfaces. In an initial investigation, a triangle was held with separate heat transfer coefficients on each half of the fin instead of a uniform heat transfer coefficient, as is typically used. It was found that thermally asymmetric fins result in longer optimal fin lengths and higher fin tip temperatures. In a similar investigation, trapezoids were investigated with asymmetric heat transfer coefficients. Likewise, it was found that the optimal fin length increases for thermally asymmetric fins with a corresponding decrease in the heat dissipated from the fin. An analysis was performed by Su and Hwang [2.28] to determine the relative error in transient one dimensional analysis of a pin fin compared with a two dimension analysis. It was determined that the relative error increased with increasing Biot number and could be as large as 10% for the conditions considered. Contreras et al. [2.29] examined the transient heat transfer from a rectangular fin with an oscillating heat rate supplied at the base. It was found that the tip temperature oscillates around an average temperature greater than that of the ambient fluid temperature.
Experimental Investigation

In this study, two fractal geometries are used to increase surface area for fins exposed to natural convection. This increase in surface area is predicted to increase the effectiveness of the fin while resulting in heat transfer per mass improvements. The first fractal set that is considered is a modified version of the Koch snowflake. The baseline case (0th iteration) and the first three iterations of can be found in Figure 2.1. The second set that is considered is the Sierpinski carpet, with the baseline case and the first three iterations for it being found in Figure 2.1 as well. The key difference between the two patterns is that the Sierpinski carpet removes material with each iteration while the modified Koch snowflake adds material with each iteration. For example, the first iteration of the Sierpinski carpet removes a square with 1/3 of the base width and the first iteration of the modified Koch snowflake adds two equilateral triangles with 1/3 of the base width. Although the Sierpinski carpet appears to have more potential, both are chosen in order to obtain a better understanding of how fractal-like geometries behave. A more detailed description of the process for performing fractal iterations is described in detail by Devaney [2.30].

![Figure 2.1. Sierpinski Carpet (left) and Modified Koch Snowflake (right)](image)

For experimentation, fractal-like fins were manufactured for the baseline case and the first three iterations out of Al-5052. The dimensions corresponding to all manufactured fins can be found in Table 2.1. An isometric view of the baseline case for the 0.1016 m width fin is
shown for both the modified Koch snowflake and Sierpinski carpet in Figure 2.2. The surface area of the modified Koch snowflake can be found using Equation 2.1 and the surface area of the Sierpinski carpet can be found using Equation 2.2.

Table 2.1

Dimensions of Experimental Fractal-Like Fins

<table>
<thead>
<tr>
<th></th>
<th>Sierpinski Carpet</th>
<th>Modified Koch Snowflake</th>
</tr>
</thead>
<tbody>
<tr>
<td>w (m)</td>
<td>0.1016</td>
<td>0.1016</td>
</tr>
<tr>
<td>t (m)</td>
<td>0.003175</td>
<td>0.003175</td>
</tr>
<tr>
<td>w (m)</td>
<td>0.0508</td>
<td>-</td>
</tr>
<tr>
<td>t (m)</td>
<td>0.0015875</td>
<td>-</td>
</tr>
<tr>
<td>w (m)</td>
<td>0.0254</td>
<td>-</td>
</tr>
<tr>
<td>t (m)</td>
<td>0.0015875</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 2.2. Isometric Baseline Views for w=0.1016 m

\[
A_s(n) = 2\left(wt + \frac{\sqrt{3}}{4}w^2\right) + \sum_{1}^{n}\left[\left(\frac{w}{3^n}\right)^2 \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{w}{3^n}\right)t\right]2^{2n-1} \tag{2.1}
\]

\[
A_s(n) = 2w^2 + 3wt - \sum_{1}^{n}8^{n-1}\left[2\left(\frac{w}{3^n}\right)^2 - 4\left(\frac{w}{3^n}\right)t\right] \tag{2.2}
\]
The change in surface area with respect to the baseline case is plotted for the first ten iterations of each fractal-like fin set in Figure 2.3. It can be observed that the modified Koch snowflake results in an increase in surface area of 58% after five iterations, with the surface area increasing each iteration. The Sierpinski carpet initially reduces surface area for each scale before reaching a minimum. However, after reaching the minimum, the surface area increases with each subsequent iteration. For the 0.0508 m and 0.1016 m fins, the ratio of surface areas are exactly equal and result in increase in surface area of 117% after five iterations. For the 0.0254 m fins, a surface area increase of 265% is obtained after five iterations. Additionally, for the 0.0508 m and 0.1016 m fins, the surface area reaches a value greater than the baseline case starting with the fourth iteration. However, for the 0.0254 m fins, the surface area reaches a value greater than the baseline case starting with the third iteration.

The mass of the modified Koch snowflake can be found using Equation 2.3 and the mass of the Sierpinski carpet can be found using Equation 2.4. The change in mass with respect to the baseline case is plotted for each fractal-like fin set in Figure 2.4. It is worth noting that while the change in surface area with respect to the baseline case is a function of fin width and thickness, the change in mass with respect to the baseline is not. The change in mass is strictly dependent on the fractal geometry chosen. The mass of the modified Koch snowflake increases each iteration before converging to a maximum value of mass increase of approximately 40%. The Sierpinski carpet, however, decreases in mass while converging to a value of zero. For comparison, after five iterations, the modified Koch snowflake results in a mass increase of 39% while the Sierpinski carpet achieves a 45% mass reduction.
Figure 2.3. Fin Area/Initial Area vs. Iteration

Figure 2.4. Fin Mass/Initial Mass vs. Iteration

\[ m(n) = \left(\frac{\sqrt{3}}{4}\right)^2 w^2 + \sum_{i=1}^{n} \left[ \frac{w}{3^n} \left(\frac{\sqrt{3}}{4}\right)^2 \right] 2^{2n-1} \rho t \]  
(2.3)
\[ m(n) = [w^2 - \sum_{i=1}^{n} 8^{n-1} \left(\frac{w}{3n}\right)^2]pt \] (2.4)

Experimental Methods

Figure 2.5 provides a simplified system drawing of the experimental test apparatus. A known heat rate was supplied to the fins through a set of ceramic resistors. The resistors used had an electrical resistance of 270 ohms with a rated temperature range of -55°C to 350°C. For the 0.1016 width fins, the base was designed such that two resistors could be clamped onto each side for a total of four resistors supplying heat. For the 0.0508 m and 0.0254 m width fins, the base was designed such that one resistor could be clamped onto each side for a total of two resistors. In order to minimize contact resistance, a high conductivity thermal paste was applied to the base of the fins in order to minimize contact resistance. The base of the fins, with the attached ceramic resistors, was insulated to minimize thermal energy losses to the surroundings. The insulation used had a thermal conductivity rated at 0.0412 W/m-K and a temperature range of -40°C to 350°C. All fins were sandblasted and painted with a flat black paint in order to provide a diffuse surface for imaging. The paint was rated to withstand temperatures up to 704°C.

![Figure 2.5. Experimental Schematic](image)

After assembling the experimental apparatus a FLIR A325 infrared thermal camera was used in each test to obtain the temperature profile across the fin. The camera has an operating
temperature range of -20°C to 120°C with emissivity correction variable from 0.01 to 1.0. Using the ExaminIR software provided by the manufacturer, an average base temperature was calculated using the average of all temperature measurements across the base of the fin. The same procedure was used to determine an average tip temperature. Sufficient time was allowed for the system to achieve steady state before any measurements were taken. Ambient temperature in the room was also recorded using a type-K thermocouple. A type-K thermocouple was also used to find the base temperature of the fin. The emissivity was found by modifying the emissivity value used by the infrared camera to ensure the measured temperature of the base matched the temperature measured by the thermocouple. Each fin was tested five times to assess measurement repeatability.

Calculations

The total heat rate was calculated from the product of the measured voltage and current. Although the heat rate can be calculated using the power dissipated by the resistors, it is necessary to know the heat lost through the insulation to the surroundings. In order to do this, the temperature differential was measured across each insulation face touching the ceramic resistors over a fixed distance to determine the heat flux. The heat flux was then multiplied by the corresponding insulation surface area to get a total heat loss rate as seen in Equation 2.5.

\[ Q_L = \sum \left( -kA_s \frac{dT}{dy} \right) \]  

(2.5)

A corrected heat rate was then calculated using Equation 2.6 to take into account losses due to the combined effects of thermal radiation and heat dissipated by the insulation such that the singular effect of convection could be studied. The emissivity used was the emissivity correlated as described earlier in the experimental methods. Once the corrected heat rate is found, the convective heat transfer coefficient can be determined from Equation 2.7.
\[ Q_c = Q - Q_L - \varepsilon \sigma A_s (T_{avg}^4 - T_\infty^4) \]  
(2.6) 
\[ Q_c = hA_s(T_{avg} - T_\infty) \]  
(2.7)

The fin efficiency was calculated using Equation 2.8 and refers to the ratio of heat transfer achieved to the heat transfer that would occur if the entire fin was at the base temperature. The fin effectiveness was calculated using Equation 2.9 and refers to the ratio of heat transfer achieved to the heat transfer that would occur if no fin was present.

\[ \eta = \frac{Q_c}{hA_s(T_b - T_\infty)} \]  
(2.8) 
\[ \varepsilon = \frac{Q_c}{hA_b(T_b - T_\infty)} \]  
(2.9)

Experimental Uncertainty

A statistical analysis was performed on the experimental data to calculate the sample standard deviation and mean of the resulting performance characteristics, including the fin effectiveness, fin efficiency, and fin effectiveness/mass. A sample standard deviation was found based on the mean value of the parameter being evaluated and the number of samples taken. Since the sample sized used corresponded to a value less than 30, a Student’s t distribution was used to provide the probability density function. Using the calculated sample standard deviation and mean, a 95% confidence interval was found based on the Student’s t distribution as described by Wheeler and Ganji [2.31]. The resulting uncertainties can be found in Table 2.2. The maximum relative uncertainties were found to be 4.5% for the fin efficiency, and 4.97% for the fin effectiveness and the fin effectiveness/mass.
Table 2.2

*Calculated Average Fin Performance Uncertainties*

<table>
<thead>
<tr>
<th>Sierpinski Carpet (w = 0.1016 m)</th>
<th>Modified Koch Snowflake (w = 0.1016 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Iteration</strong></td>
<td><strong>0</strong>  <strong>1</strong>  <strong>2</strong>  <strong>3</strong></td>
</tr>
<tr>
<td><strong>Fin Efficiency</strong></td>
<td><strong>0.01</strong>  <strong>0.02</strong>  <strong>0.01</strong>  <strong>0.02</strong></td>
</tr>
<tr>
<td><strong>Fin Effectiveness</strong></td>
<td><strong>0.67</strong>  <strong>1.43</strong>  <strong>0.84</strong>  <strong>1.05</strong></td>
</tr>
<tr>
<td><strong>Fin Effectiveness/Mass, kg⁻¹</strong></td>
<td><strong>7.6</strong>  <strong>18.3</strong>  <strong>12.1</strong>  <strong>17.0</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sierpinski Carpet (w = 0.0508 m)</th>
<th>Sierpinski Carpet (w = 0.0254 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Iteration</strong></td>
<td><strong>0</strong>  <strong>1</strong>  <strong>2</strong>  <strong>3</strong></td>
</tr>
<tr>
<td><strong>Fin Efficiency</strong></td>
<td><strong>0.04</strong>  <strong>0.02</strong>  <strong>0.01</strong>  <strong>0.01</strong></td>
</tr>
<tr>
<td><strong>Fin Effectiveness</strong></td>
<td><strong>3.04</strong>  <strong>1.11</strong>  <strong>0.55</strong>  <strong>0.77</strong></td>
</tr>
<tr>
<td><strong>Fin Effectiveness/Mass, kg⁻¹</strong></td>
<td><strong>277.0</strong>  <strong>114.1</strong>  <strong>63.4</strong>  <strong>99.7</strong></td>
</tr>
</tbody>
</table>

Results

Fin effectiveness and fin efficiency values were calculated for each iteration using the previously described equations and are found tabulated in Table 2.3. The efficiencies can be found plotted in Figure 2.6 and Figure 2.7. It is important to note that although for all figures shown in this paper there is a linear line connecting points, this does not imply functional behavior.

Table 2.3

*Calculated Average Fin Performance Characteristics*

<table>
<thead>
<tr>
<th>Sierpinski Carpet (w = 0.1016 m)</th>
<th>Modified Koch Snowflake (w = 0.1016 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Iteration</strong></td>
<td><strong>0</strong>  <strong>1</strong>  <strong>2</strong>  <strong>3</strong></td>
</tr>
<tr>
<td><strong>Fin Efficiency</strong></td>
<td><strong>0.89</strong>  <strong>0.88</strong>  <strong>0.85</strong>  <strong>0.82</strong></td>
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<tr>
<td><strong>Fin Effectiveness</strong></td>
<td><strong>58.85</strong>  <strong>52.74</strong>  <strong>48.54</strong>  <strong>50.40</strong></td>
</tr>
<tr>
<td><strong>Fin Effectiveness/Mass, kg⁻¹</strong></td>
<td><strong>670.1</strong>  <strong>675.7</strong>  <strong>699.6</strong>  <strong>817.1</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sierpinski Carpet (w = 0.0508 m)</th>
<th>Sierpinski Carpet (w = 0.0254 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Iteration</strong></td>
<td><strong>0</strong>  <strong>1</strong>  <strong>2</strong>  <strong>3</strong></td>
</tr>
<tr>
<td><strong>Fin Efficiency</strong></td>
<td><strong>0.91</strong>  <strong>0.92</strong>  <strong>0.91</strong>  <strong>0.87</strong></td>
</tr>
<tr>
<td><strong>Fin Effectiveness</strong></td>
<td><strong>61.15</strong>  <strong>56.00</strong>  <strong>53.11</strong>  <strong>54.13</strong></td>
</tr>
<tr>
<td><strong>Fin Effectiveness/Mass, kg⁻¹</strong></td>
<td><strong>5570.6</strong>  <strong>5739.4</strong>  <strong>6123.9</strong>  <strong>7021.8</strong></td>
</tr>
</tbody>
</table>
Figure 2.6 shows the comparison between the Sierpinski carpet \((w = 0.1016 \text{ m})\) and the modified Koch snowflake \((w = 0.1016 \text{ m})\). The Sierpinski carpet efficiency decreases with each iteration and can be explained by the fact that the temperature differential across the face of the fin increases with each subsequent iteration. Also, the first iteration of the Sierpinski carpet results in a large increase in thermal resistance due to the large square perforation created. However, due to a much smaller temperature differential across the face of the fin, the efficiency of the modified Koch snowflake remains relatively constant with each iteration. The first iteration results in a slight increase in efficiency with the second iteration resulting in a slight decrease. The third iteration results in another slight increase. This can be explained by the similar temperature differential across the face of the fin for each iteration.

Figure 2.6. Average Fin Efficiency vs. Iteration

Figure 2.7 shows the comparison between all three scales of the Sierpinski carpet. As expected, the 0.0254 m width fins have the highest efficiencies with the 0.1016 m width fins
having the lowest. This is due to the smaller temperature differential across the face of the fins due to the relative size difference of the 0.0254 m width fin in comparison with the 0.1016 m width fin. It is also seen that each fin scale results in a decline in fin efficiency across the tested iterations as noted previously with the 0.1016 m width fin.

![Figure 2.7. Average Fin Efficiency vs. Iteration](image)

The values for fin effectiveness can be found plotted in Figure 2.8 and Figure 2.9. Figure 2.8 shows the comparison between the Sierpinski carpet (w = 0.1016 m) and the modified Koch snowflake (w = 0.1016 m). For the Sierpinski carpet, the fin effectiveness follows the same trend as the change in relative surface area of each iteration. The effectiveness decreases for the first two iterations due to a reduction in surface area from the previous iteration. However, the third iteration results in an increase from the second iteration due to an increase in surface area. While the effectiveness of the third iteration increases from the second iteration, it does not surpass that of the baseline case as its surface area is still smaller than the baseline. The modified Koch
snowflake also follows the same trend as the change in relative surface area of each iteration. It results in an increase in effectiveness with each iteration. The first iteration results in a large relative increase in effectiveness with each subsequent iteration resulting in a smaller relative increase of effectiveness and ending with a value 44.8% greater than the baseline case effectiveness.

Figure 2.8. Average Fin Effectiveness vs. Iteration

Figure 2.9 shows the comparison between all three scales of the Sierpinski carpet. As expected, due to an identical area vs. initial area ratio at each iteration, the effectiveness of the 0.0508 m width and 0.1016 m width fins are comparable. Like the 0.1016 m width fins, the 0.0508 m width fins resulted in a decrease in effectiveness for the first two iterations followed by an increase in effectiveness with the third iteration. Similarly, the 0.0254 m width fins followed the same trend as the change in relative surface area of each iteration. Unlike the other two scales of the Sierpinski Carpet, the 0.0254 m width fins only decrease in effectiveness with the first
iteration. The second and third iterations result in increases of effectiveness. It is important to note that for the third iteration, the effectiveness increases to a value 11.4% greater than that of the baseline case effectiveness. This is critical as it can be seen that for both fractal patterns, iterations can be used to increase the effectiveness beyond that of the baseline itself.

![Graph of Average Fin Effectiveness vs. Iteration](image)

**Figure 2.9. Average Fin Effectiveness vs. Iteration**

Although the fin effectiveness and fin efficiency are key parameters, another reason for investigating the concept of fractal-like fins is the potential for achieving high performance on a fin unit mass basis. In order to compare the performance in this light, the effectiveness per mass was calculated simply by dividing the calculated effectiveness by the mass of each iteration and can be found plotted in Figure 2.10 and Figure 2.11. Figure 2.10 shows the comparison between the Sierpinski carpet (w = 0.1016 m) and the modified Koch snowflake (w = 0.1016 m). As can be seen, the Sierpinski carpet results in increases in effectiveness per mass with each iteration. This increase becomes continuously larger and results in a 22% increase in effectiveness per
mass above the baseline case after the third iteration. The modified Koch snowflake in comparison has a slight decrease in effectiveness per mass with the first iteration due to the large increase in mass required to perform the first iteration. However, with the second and third iterations, the effectiveness per mass increases with a 6% increase above the baseline case after the third iteration. Although the Sierpinski carpet is found to be more effective on a mass basis with each iteration, the modified Koch snowflake would require at least a second iteration to achieve greater performance than the baseline case.

Figure 2.10. Average Fin Effectiveness/Mass vs. Iteration

Figure 2.11 shows the comparison between all scales of the Sierpinski carpet. The 0.0508 m width fins compare similarly to the 0.1016 m width fins as expected and result in a 26% increase in effectiveness per mass above the baseline case. The 0.0254 m width fins result in a continuously larger increase in effectiveness per mass with each iteration as seen with the
previous scales. They differ from the previous two scales though as the third iteration results in a 59% increase in effectiveness per mass above the baseline case.

![Figure 2.11. Average Fin Effectiveness/Mass vs. Iteration](image)

**Figure 2.11. Average Fin Effectiveness/Mass vs. Iteration**

**Experimental Correlation**

In order to better quantify the behavior of fractal-like fins, an experimental correlation of the data was obtained using the Buckingham Pi Theorem, as described in Fox & McDonald [2.32]. It was determined that four dimensionless groups could be used to correlate fin effectiveness as seen in Equation 2.10.

$$
e = C N u^{d_1} \theta^{d_2} \Lambda_n^{d_3} I^{d_4}$$

(2.10)

The constants were found by minimizing the average of the error between the predicted fin effectiveness and the experimentally calculated fin effectiveness. Separate correlations were determined for the Sierpinski carpet and the modified Koch snowflake with the corresponding
constants presented in Table 2.4. For the Sierpinski carpet correlation, shown in Figure 2.12, a maximum error of 2.36% was found with an average error of 1.1%. For the modified Koch snowflake correlation, shown in Figure 2.13, a maximum error of 1.4% was found with an average error of 0.77%.

Table 2.4

<table>
<thead>
<tr>
<th>Experimental Fin Effectiveness Correlation Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sierpinski Carpet</strong></td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>2.395</td>
</tr>
<tr>
<td><strong>Modified Koch Snowflake</strong></td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>1.825</td>
</tr>
</tbody>
</table>

Figure 2.12. Experimental Fin Effectiveness Correlation
Conclusions

An investigation into the use of fractal geometries to enhance fin performance was conducted. Based upon the observed results of the fractal patterns the following statements can be made:

- Effectiveness of the tested fractal-like fins behaves proportionally to the available surface area and can be seen to increase by up to 11.4% for the third iteration of the Sierpinski carpet and up to 44.8% for the third iteration of the modified Koch snowflake.

- Effectiveness per mass increases by larger steps with higher order iterations and can be seen to increase by up to 59% for the third iteration of the Sierpinski Carpet and up to 6% for the third iteration of the modified Koch Snowflake.

- For the modified Koch snowflake, increased effectiveness is available after the first iteration and with each subsequent iteration.
- For the Sierpinski carpet, increased effectiveness is only available with the third iteration for the 0.0254 m width fins, although further increases are expected with additional iterations.

- Experimental correlations were developed to characterize the fin effectiveness with a maximum error of 2.36% for the Sierpinski Carpet and a maximum error of 1.4% for the modified Koch snowflake.

As seen by the results, the effectiveness of a fractal-like fin can be predicted for at least the first three iterations by investigating the relative surface area compared with the baseline case. Although the trends predict that a fourth and fifth iteration of the Sierpinski carpet would result in greater effectiveness for the 0.0508 m width and 0.1016 m width fins, more testing is needed to confirm this.

Acknowledgements

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References


CHAPTER 3

NATURAL CONVECTION HEAT TRANSFER FROM FRACTAL-LIKE FINS*

Abstract

The results of a computational investigation into the performance of fractal-like fins under conditions where natural convection is the sole heat transfer mechanism are presented. Fractal patterns such as the Sierpinski carpet have previously been shown to increase surface area significantly, compared to classical geometries, while also reducing mass. This study examines fin performance for a baseline case (a square fin) relative to the first four fractal iterations of fins inspired by the Sierpinski carpet fractal pattern. To evaluate thermal performance, constant heat flux conditions were applied to the base of the fins. Geometric fin parameters, surface heat flux, fin scale, and fin orientation were varied to understand their role with regard to fin effectiveness and fin efficiency. Fin effectiveness was found to be greatly increased through the use of fractal-like fins due to the increase in surface area per fin volume. Fin effectiveness per unit mass was also found to be improved significantly, which could lead to use in applications where the fin mass is a selection criterion.

Introduction

The removal of excess heat generated by electrical equipment is an essential task to ensure optimal performance and to reduce the risk of component failures. Typically, extended surfaces in the form of heat sinks are attached to locations where this excess heat must be

dissipated in an effort to provide an increase in surface area. Applications involving extended surfaces range from electronics, engines, industrial equipment, and a variety of other mechanical devices. Although the overall goal is often to maximize the heat transfer, other goals include minimization of fin volume and fin mass. This can be a problem as convective heat transfer is proportional to the surface area available.

Fractals, as described by Mandelbrot [3.1], are geometric patterns that are repeated at infinitely smaller scales to produce irregular shapes, surfaces, or designs that are not described by classical geometries. They are more efficient at filling space than classical geometric shapes and their characteristics include self-similarity, non-integer dimension, and scale invariance. It has previously been proposed that fractal geometries can lead to significant increases in surface area when compared to standard fin geometries [3.2]. Utilizing fractal geometries for fins can result in a reduction in fin volume to maintain a desired fin surface area or a reduction in mass if fin volume is maintained. Both of these cases could be beneficial to commercial and industrial applications where an increase in convective heat transfer or a reduction in fin mass is desirable.

Fractals have been used in many areas related to engineering applications. The use of fractals in engineering has wide ranging applications and has been demonstrated with previous works [3.3-3.12]. Fractal patterns have been shown to create effective distribution systems for use in applications involving chemical reactions [3.3] as well as to increase the capacitance per area for parallel plate capacitors [3.4]. Fractal patterns are commonly used to compress images to utilize less memory than standard techniques, while maintaining image quality [3.5]. An increase in the water repellence of a material has been shown to be created by arranging hydrophobic surfaces in a fractal pattern [3.6]. Additionally, fractals are commonly used to increase the performance of radio and cell phone antenna while reducing the volume used [3.7,3.8].
Although fractals have been previously shown to increase the performance of antennas, recently published studies have examined the ability of fractals to increase heat transfer. Casanova et al. [3.9] demonstrated that when compared to a plate fin heat sink, a fractal heat sink antenna could improve radiation efficiency while decreasing the thermal resistance with fractal iterations. Van Der Vyver [3.10] computationally studied a tube-in-tube heat exchanger, with the inner tube shaped in the pattern of a quadratic Koch island. Heat transfer within the heat exchanger was found to increase by a factor of two for each fractal iteration and the findings were confirmed by Meyer and Van De Vyver [3.11] through a follow up experimental investigation. Lee et al. [3.12] developed a solution to minimize temperature induced error in the measurement readings of a laser interferometer. By utilizing a third order Hilbert curve, the heat transfer area inside the interferometer was increased without an increase in volume.

Other uses of fractals in heat transfer have been the enhancement of surface heat transfer by utilizing fractal surface geometries. Adrover [3.13] investigated the effects of fractal surfaces on the thermal boundary layer and noted that the heat transfer is proportional to the surface’s fractal dimension. Blyth and Pozrikidis [3.14], as well as Brady and Pozrikidis [3.15], investigated the effects of fractal surface modifications on heat transfer by examining fractal patterns and the number of iterations. It was found that although an infinite number of fractal iterations can result in an infinite increase in surface area, improvements in heat transfer typically converge around five iterations depending on the fractal pattern chosen.

Similar to the Sierpinski carpet fractal pattern utilized in this paper, perforations of fins have been studied in an attempt to increase heat transfer or reduce weight. Sahin and Demir [3.16,3.17] investigated the heat transfer enhancement of circular and square pin fins by inserting perforations into the body of the fin. It was determined that perforations could produce increases
in heat transfer per pumping power. Al-Essa et al. [3.18-3.20] investigated the effects of square, rectangular, and triangular perforations into rectangular fins under natural convection. For all conditions tested, it was found by removing material through perforations, the heat transfer per unit mass could be increased. Further evaluation found that the optimal orientation of fins with perforations was found to vary, depending on fin geometric parameters. Akyol and Bilen [3.21] investigated the increased pumping power required to produce increased thermal performance for an aligned array of fins with hollow channels running down the width. It was observed that heat transfer per pumping power increased with perforations. Further studies were investigated by Sara et al. [3.22] for fins with perforations inserted into the thickness of the fin for varying inclination angle of perforations and varying perforation sizes. Increases in heat transfer per pumping power were achieved, with increases of up to twenty percent observed when compared to fins without perforations. Shaeri et al. [3.23-3.25] investigated the thermal performance for fins with perforations inserted into fins tangential and normal to turbulent flow. For perforations inserted tangential to the flow, it was found that although minimal heat transfer increases were obtained, a large decrease in the pumping power was achieved. For perforations inserted normal to the flow, it was found that a large increase in heat transfer was achieved as well as a large decrease in pumping power. The decrease in pumping power was the result of a reduction in the turbulent wake behind the fins due to the introduction of perforations. An investigation by Elsahfei [3.26] found that a combination of a hollow pin fin with perforations can increase heat transfer when compared with a solid pin fin with the same diameter. Although not a perforation, an investigation in the thermal performance of a fin with a semicircular cut at the tip of the fin was evaluated by Prasad and Gupta [3.27]. By removing a semicircular piece, an increase in the
fin effectiveness per unit mass was achieved for any radii cuts, with fin effectiveness and fin efficiency being increased for a range of radii cuts.

The performance of fins operating under natural convection conditions has been extensively studied. Yüncü and Anbar [3.28] investigated the parameters of fin height and spacing for given heat conditions to find an optimal plate-fin heat sink design. It was found that optimal spacing between fins decreases with increasing fin height. Razelos and Georgiou [3.29] investigated the performance of longitudinal, annular, and pin fins based on the fin effectiveness. It was found that an effectiveness of 10 corresponded to the minimum value warranting the use of fins. An optimized plate-fin heat sink based on specified heat sink mass and volume was developed by Bar-Cohen et al. [3.30] and presented optimal aspect ratios and optimal fin spacing for the inputs. Although heat sinks are typically modeled as having an isothermal base, Van De Pol and Tierney [3.31] developed a Nusselt number correlation for a plate fin heat sink in which the base had a variable temperature across the surface. Comparison of scaled behavior was investigated when Mahmoud et al. [3.32], who compared optimal dimensions for micro-scale heat sinks with macro-scale heat sinks. It was found that results for macro-scale designs are not applicable to micro-scale heat sink design. The effect of the inclination angle of fins in a heat sink was studied by Fujii and Imura [3.33] and a corresponding Nusselt number was developed. It was noted that increasing inclination resulted in higher separation of the boundary layer. Sunden [3.34] noted that as the convection-conduction parameter increased, the section of the fin where the buoyant forces are large became restricted to a smaller region near the base of the fin.

Patterns similar to fractals have been utilized previously to enhance heat transfer in the form of constructal fins. Bejan and Almogbel [3.35] showed that constructal fins could be shown to minimize thermal resistance for a point-to-volume fin by using branching geometries shaped
with T shaped branches. In addition Lorenzini [3.36] minimized thermal resistance using a T-Y branched shape with a cavity in between the initial branch and found that its effectiveness was 32% higher than any previously designed constructal fins. Zimparov [3.37] extended the constructal patterns to flow paths in parallel and counter flow heat exchangers and found that optimal branching systems varied depending on the pumping power.

Increases in the available surface area of fins can be achieved with corresponding reduction of fin mass by utilizing fractal geometries. In a previous study, Dannelley and Baker [3.2] experimentally discovered that the use of fractal-like fins could increase fin effectiveness and fin effectiveness per unit mass for natural convection heat transfer. The results presented here will further examine the thermal performance of fractal-like fins and their potential to increase heat transfer per unit mass by expanding upon previous experimental results using computational models.

**Computational Method**

Simulations were performed using a commercially available computational fluid dynamics software package. The three-dimensional form conservation of mass, momentum, and energy equations governing the fluid region are given as:

\[
\frac{\partial}{\partial x_i} (\rho u_i) = 0 \tag{3.1}
\]

\[
\frac{\partial}{\partial x_i} (\rho u_i u_j) + \frac{\partial p}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) - \rho g_i \tag{3.2}
\]

\[
\frac{\partial}{\partial x_i} (\rho u_i c_p T) = \frac{\partial}{\partial x_i} \left( k \frac{\partial T}{\partial x_i} \right) \tag{3.3}
\]

Thermophysical properties of air, which is assumed an ideal gas, are evaluated as a function of fluid temperature. The governing equation for the solid region, i.e. the fractal-like fin, is given as:
\[ \nabla^2 T = 0 \] (3.4)

A constant heat flux boundary condition was applied to the base of the fractal-like fin, i.e.:

\[ q'' = -k \frac{\partial T}{\partial x_n} \bigg|_{S_b} \] (3.5)

The governing equations are solved using a cell-centered finite volume method with all values of calculated parameters and physical properties stored in the center of the mesh cell [3.38-3.40]. Spatial derivatives are approximated with implicit difference operators and are second order accurate. Second order accurate implicit approximations for fluxes are obtained using the QUICK approximation [3.41] and the TVD method [3.42]. A SIMPLE-like correction is used to solve for the pressure using transformations of the discretized mass and momentum equations accounting for velocity boundary conditions [3.43].

Validation/Verification

Validation of the computational model was performed using previous experimental data and theoretical solutions. For the initial validation, the zeroth and first three fractal iterations of the Sierpinski carpet were modeled for an aluminum fin with width of 0.1016 m and thickness of 0.3175 cm. The applied heat rate at each iteration corresponded to the calculated convective heat rate from experimental results [3.2]. The comparison of the difference between experimentally and computationally calculated fin effectiveness can be seen in Figure 3.1. The maximum difference between the solutions is found to be 12%. However, as can be seen, the trend of fin effectiveness is the same across iterations. A source of error between the model and experimental results can be attributed to the radiation assumption utilized in the experimental results. In order to isolate the effects of natural convection for experimentation, the convective heat rate was subtracted from the overall heat rate by calculating the total radiative heat rate based on the total
surface area. This assumption was chosen for simplicity and does not account for radiative exchange between surfaces due to perforations in the fin. Another source of error could be the experimental assumption of a constant heat transfer coefficient. The computational model accounted for variable heat transfer coefficients across the fin surface. A second validation was performed using previous benchmark solutions from Incropera et al. [3.44]. The calculated fin efficiency for the zeroth iteration fin corresponding to the previously mentioned dimensions was found to be within 3.8% of the theoretical value. Error from this validation can be attributed to the one dimensional assumption for the theoretical model and the three dimensional analysis utilized in the computational model.

![Figure 3.1. Fin Effectiveness vs. Iteration](image-url)

Figure 3.1. Fin Effectiveness vs. Iteration

A spatial grid refinement was performed for the case with the most complex geometry, which corresponding to the fourth iteration of the Sierpinski carpet with a width to thickness ratio of 64. For grid independence, the number of cells was incrementally doubled and the fin
effectiveness and fin efficiency were calculated until a change of less than 1% was achieved. A total number of cells of approximately 2,000,000 was found to be sufficient to achieve the criterion.

Performance Characteristics

The primary means for comparison of fin performance in this paper are fin effectiveness, fin effectiveness per unit mass, and fin efficiency. Fin efficiency represents the ratio of heat transfer dissipated by the fin to the heat transfer that would be dissipated if the entire fin was at the base temperature and was calculated using:

\[ \eta = \frac{Q}{hA_b(T_b - T_\infty)} \]  

(3.6)

Fin effectiveness represents the ratio of heat transfer dissipated by the fin to the heat transfer that would be dissipated by the base area if no fin was present and is calculated using:

\[ \varepsilon = \frac{Q}{hA_b(T_b - T_\infty)} \]  

(3.7)

Fin effectiveness per unit mass will be examined in order to investigate the ability of fractal-like fins to increase thermal performance for applications where reduced mass is desirable.

Analysis

In this study, the thermal performance of fractal-like fins inspired by the Sierpinski carpet was examined. This geometry is studied to give added insight and expand upon previous experimental results. Figure 3.2 displays dimetric views of the zeroth and first four iterations of the fractal-like fins inspired by the Sierpinski carpet for a width/thickness ratio of 32. As seen, the zeroth iteration is simply a square. The first iteration results in the removal of a square with 1/3 of the base width. Each subsequent iteration results in the removal of 8n-1 squares with 1/3n base width. By removing square sections of the fin, the mass of the fin is reduced with each
iteration. Although the surface area of the fin is initially reduced due to the loss of fin face area, the rim area can be found to increase the total surface area after performing enough fractal iterations. For example, the fourth iteration of a Sierpinski carpet fractal-like fin with a width/thickness ratio of 32 will result in a decrease in mass of 38% and an increase in surface area of 23%. The process for determining the mass and surface area is described in further detail by Dannelley and Baker [3.2].

Figure 3.2. Sierpinski Carpet Iterations

Results

In order to quantify fin performance, fractal-like fins are modeled for the zeroth iteration and first four iterations for all geometric parameters, heat rates, and orientations considered. A baseline case was established that corresponds to an aluminum fin with a fin base width of 10.16 cm, fin base thickness of 0.3175 cm, and base heat flux of 15 kW/m$^2$. The effects of the width/thickness ratio were evaluated by holding the base width constant and varying the thickness such that the width/thickness ratio ranges from 8 to 64. The fin base heat flux was varied from 15 - 120 kW/m$^2$ to understand the thermal performance as a function of the heat rate. The scaled thermal performance was studied by examining fins with base widths on the order of 0.01 m, 0.1 m, and 1 m. Although not very practical, a 1 m fin was examined for a scaling analysis. Finally, the orientation of the fin with respect to the direction of gravity was evaluated by using horizontal and vertical fin configurations.
Figure 3.3 presents the fin effectiveness as a function of iteration and width/thickness ratio. The thermal performance for varying fin width/thickness ratios was evaluated by holding the width constant and varying the thickness. An increasing width/thickness ratio results in a progressively thinner fin. For smaller values of width/thickness ratio examined, the fin effectiveness results in slight decreases for the first iteration before increasing with subsequent iterations. For larger values of width/thickness ratio examined, the fin effectiveness results in decreases with the first three iterations before increasing with the fourth iteration. For a fin with width/thickness ratio of 8, the relative increase in fin effectiveness for the fourth iteration is 152% compared to the zeroth iteration, while the relative increase in fin effectiveness for a width/thickness ratio of 64 is -14% compared to the zeroth iteration.

![Figure 3.3. Fin Effectiveness vs. Iteration](image)

The fin effectiveness variation with width/thickness ratio can be explained by the relative change in surface area with each iteration. As the fin width/thickness ratio is decreased, the effects of perforations caused by performing iterations of the Sierpinski carpet become more
pronounced. For thicker fins, perforations result in larger increases of the total surface area due to increased rim surface area. Rim surface area is the area created due to perforations. Additionally, an increase in heat transfer coefficient is achieved near the edges created from perforations with higher order iterations. Figure 3.4 presents the heat transfer coefficient across the centerline of the fin for the zeroth and third iteration for a width/thickness ratio of 32. The centerline across the face of the third order iteration is also visualized in Figure 3.4 where a zero value of the y-axis corresponds to the fin base. It should be noted that since Figure 3.4 presents the heat transfer coefficient across the centerline of the fin face, there is discontinuity for the third iteration due to perforations. It can be observed that major spikes in the heat transfer coefficients are present near edges with the average heat transfer coefficient of the third iteration 79% greater than that of the zeroth iteration. Similarly, an increase in the average heat transfer coefficient of 108% is obtained for a width/thickness ratio of 64 and an increase of 8% is achieved for a width/thickness ratio of 8.

![Figure 3.4. Heat Transfer Coefficient vs. Centerline Distance](image)

Figure 3.4. Heat Transfer Coefficient vs. Centerline Distance
Figure 3.5 presents the fin effectiveness per unit mass as a function of iteration and width/thickness ratio. As the relative change in mass with iteration is a function of the fractal pattern chosen, the fin effectiveness per unit mass is similar to the fin effectiveness. For a fin with width/thickness ratio of 8, the relative increase in fin effectiveness per unit mass for the fourth iteration is 303% compared to the zeroth iteration. The relative increase in fin effectiveness for a width/thickness ratio of 64 is 37% compared to the zeroth iteration. As a result of fin mass decreasing by 11% with each fractal iteration, fin effectiveness per unit mass increases with iteration for all width/thickness ratios tested.

![Figure 3.5. Fin Effectiveness/Mass vs. Iteration](image)

Figure 3.6 presents the fin efficiency as a function of iteration and width/thickness ratio. As expected, the fin efficiency at each iteration increases with decreasing width/thickness ratio and decreases with increasing iteration. For fins with large surface areas, the temperature gradients are naturally larger, which decreases the fin efficiency. A noteworthy result was
observed for the fin efficiency of the fourth iteration for a width/thickness ratio of 8. The fin efficiency decreased to a value close to that of the fin efficiency for a width/thickness ratio of 32. This result shows that for a width/thickness ratio of 8, the thermal performance is approaching its limit, corresponding to behavior previously observed [3.12-3.14], and improvement will not increase greatly with future iterations. Similar results would be expected for the other width/thickness ratios with subsequent iterations beyond those examined.

![Fin Efficiency vs. Iteration](image)

*Figure 3.6. Fin Efficiency vs. Iteration*

The effect of varying the surface heat flux was also evaluated. Figure 3.7 presents the fin effectiveness as a function of the iteration and base heat flux. It can be seen that for all heat fluxes examined, the fin effectiveness and at each iteration follow the same trend as the baseline case. A heat flux of 15 kW/m$^2$ resulted in a fin effectiveness of 73.6 after four iterations while a heat flux of 120 kW/m$^2$ resulted in a fin effectiveness of 70.9, a reduction of 3.7%. Similarly, the fin efficiency after the fourth iteration was decreased by 3.3%. The fin effectiveness can be
improved even for a heat flux as large as 120 kW/m², although for heat fluxes on that order forced convection would be needed as a flux of this magnitude resulted in temperatures on the order of 500 K. This observation demonstrates that the performance of fractal-like fins is not adversely affected by the heat flux.

![Figure 3.7. Fin Effectiveness vs. Iteration](image)

The performance of fractal-like fins at multiple scales was examined to determine the usefulness of fractal-like fins for varying scaled applications. In addition to the previously mentioned fin effectiveness for fins with widths on the order of 0.1 m, fins with base width of 0.01 m and 1 m were considered. Figure 3.8 presents the fin effectiveness as a function of iteration and scale. Fin effectiveness is found to decrease with increasing size and the trend in fin effectiveness for the 0.01 m and 0.1 m width fins is almost identical. However, for the 1 m width fins, the relative increase in fin effectiveness at each iteration is reduced when compared with the smaller width fins. A relative increase in fin effectiveness of 19% and 14% is achieved for fin
widths of 0.01 m and 0.1 m compared to the zeroth iteration, while a relative decrease in fin effectiveness of 7% is achieved for a fin width of 1 m compared to the zeroth iteration. Similar to previous results it was observed that an increase in the average heat transfer coefficient along the centerline of 116% was achieved for the 0.01 m width fins. However, an increase of only 56% was achieved for the 1 m width fins, which demonstrates that the effects of edges are less significant as fin size increases. Figure 3.9 presents the fin efficiency as a function of iteration and scale. Fin efficiency is found to increase with decreasing fin size, with larger decreases in fin efficiency observed with iterations for larger fins. The decrease in fin efficiency can be explained by the increase in the temperature gradient between the fin base and tip with increasing size.

Figure 3.8. Fin Effectiveness vs. Iteration
In a previous experimental study \[3.2\], the following correlation was developed for the fin effectiveness:

\[
\varepsilon = 2.395\text{Nu}^{0.052}\theta^{-0.458}N_{n}^{1.057}T^{-0.907}
\] (3.8)

All calculated values of fin effectiveness for varying width/thickness ratio, heat rate, and scales were used in conjunction with the experimental correlation to assess its application to the additional parameters examined in this study. Figure 3.10 presents the predicted fin effectiveness for the computational data compared to calculated fin effectiveness. The average difference between the predicted fin effectiveness and calculated fin effectiveness was found to be 2.5% while the maximum difference was found to be 8.4%. The largest discrepancies between the calculated and predicted fin effectiveness were obtained for the fourth iteration of fins with width/thickness ratios of 8 and 64. These differences can be attributed to the experimental data.
used for the experimental correlation consisting only of width/thickness ratios of 16 and 32 with a maximum number of only three fractal iterations.

\[ \varepsilon = 2.715N u^{-0.005} \theta^{-0.477} A \tilde{R}_{n}^{0.933} \tau^{0.933} \]  

(3.9)

As seen in Figure 3.11, the improved correlation resulted in an average difference between the predicted and calculated fin effectiveness of 0.84% with a maximum error of 3.04%. The improvement in the correlation is a direct result of integrating additional geometric parameters, scales, heat fluxes, and fourth fractal iteration.
Previous studies [3.18-3.20] indicated that the effect of fin orientation with respect to gravity was an important factor. It was observed that for thick fins with perforations, the fin effectiveness could be improved by arranging the fin horizontal when compared to gravity [3.18-3.20]. In order to investigate the effects of orientation on fractal-like fins, the fin effectiveness was evaluated for a heat flux of 15 kW/m² with width/thickness ratios of 8, 16, and 32. Figure 3.12 presents the fin effectiveness as a function of width/thickness, orientation, and iteration. The letters H and V correspond to horizontal and vertical arrangements with respect to gravity. It can be observed that for small values of iteration and width/thickness ratio, the fin effectiveness is independent of the orientation. However, the vertical arrangement begins to slightly outperform the horizontal arrangement at higher order iterations. It was observed that an increase in the average heat transfer coefficient along the centerline of 67% was achieved for the horizontal fins compared to a 79% increase for the vertical orientation. This demonstrates that the edge effects
from perforations are slightly less beneficial for the horizontal arrangement, with growing difference in the fin effectiveness expected for higher order iterations. For comparison, the difference in fin effectiveness for vertical and horizontal arrangement is 2% for a width/thickness ratio of 8 while the difference is 4% for a width/thickness ratio of 32. Although the difference between the two orientations increases with thinner fins, the difference in fin effectiveness for orientation is not significant.

![Figure 3.12. Fin Effectiveness vs. Iteration](image)

**Conclusions**

An investigation into the use of fractal-like geometries to enhance natural convective fin effectiveness was conducted. Once the computational model was validated and verified, an expansion of previous experimental results was undertaken to examine fin effectiveness and fin
efficiency. Based upon the observed results of the fractal-like fins, the following conclusions may be drawn:

- Fin effectiveness per unit mass is improved with iteration and increases by at least 37% after four iterations for the conditions examined.
- Greater improvements in fin effectiveness per unit mass are achieved for thicker fins, with a relative increase of 303% for a width/thickness ratio of 8 after four iterations.
- Heat flux does not adversely affect the thermal performance, as the fin effectiveness is only reduced by 3.7% for an increase in heat flux of 800%.
- Fin effectiveness was observed to increase with decreasing fin width, with fin effectiveness improved by 19% for fin width of 0.01 m compared to a decrease of 7% for fin width of 1 m.
- An experimental correlation for the fin effectiveness was improved by incorporating additional data from the computational investigation. This resulted in a maximum difference of 3.04% and an average difference of 0.84% between calculated and predicted fin effectiveness.

Effectiveness of fractal-like fins was found to be relatively independent of orientation with respect to gravity for the conditions considered as the maximum difference in effectiveness for orientation was 4%.

As observed, the fin effectiveness and the fin effectiveness per unit mass can be significantly improved through the use of fins modeled after the Sierpinski carpet fractal pattern. This can be attributed to increases in surface area per volume as well and reduction of fin mass.
References


CHAPTER 4

RADIANT FIN PERFORMANCE USING FRACTAL-LIKE GEOMETRIES*

Abstract

The results of a computational study into the thermal performance of thermally radiating fractal-like fins are presented. Previous experimental studies have shown that fractal patterns increase the heat transfer surface area while simultaneously reducing mass. Two fractal patterns were used for comparison, the modified Koch snowflake and the Sierpinski carpet. For an isothermal base fin radiating to free space, the fin effectiveness and fin efficiency are presented for the zeroth and first four fractal iterations in order to quantify the performance. Emissivity, width/thickness ratio, base temperature, and fin material were varied to better understand their impact on the performance of fractal-like fins. Based upon the observed results, fractal-like fins greatly improve the fin effectiveness per unit mass. In certain cases, fin effectiveness per unit mass was found to increase by up to 46%. As the cost of access to space is significant, this reduction in mass could lead to savings for spacecraft thermal management applications.

Introduction

The dissipation of waste heat is a critical function for spacecraft. Waste heat generated onboard a satellite or any other orbiting equipment must either be converted into useful energy or rejected to its surroundings. This can be difficult however, as the only mode of heat transfer available for removing this waste thermal energy in a space environment is thermal radiation. A common technique for heat transfer from equipment in space is to utilize flat-plate radiators

* Note. This chapter is based on "Radiant Fin Performance Using Fractal-Like Geometries," by D. Dannelley, and J. Baker, accepted for publication in the Journal of Heat Transfer, January 9, 2013.
attached to the body of the spacecraft and/or additional panels that can be employed once the object is in orbit [4.1]. Body-mounted flat-plate radiators are often used to accommodate applications that have individual temperature constraints separate from other components [4.1]. It is desirable for these extended surfaces to utilize the least mass of material while maximizing heat rejection due to the high costs associated with the mass of the payload. Since thermal radiation heat transfer is a function of the available surface area, this can be difficult to achieve without the use of higher volume/mass extended surfaces.

There have been numerous studies to examine the radiant performance of fins based on minimizing weight and accounting for irradiation between fins. Krikkis et al. [4.2] investigated the optimization of rectangular and triangular fins and found that for opening angles between fins less than 180°, the optimal properties for fin height and thickness are highly dependent on the emissivity of the fin. Kumar et al. [4.3] performed a mass based optimization to find the optimal number of fins for a plate fin heat sink. A sharp initial increase in heat transfer was found until the optimal number of fins was reached, followed by a gradual decline with additions of further fins. Sparrow et al. [4.4-4.6] investigated multiple radiation heat transfer solutions involving mutual irradiation between elements. In an examination of the performance of a plate fin heat sink, it was determined that it is important to optimize the combined performance of the base as well as the fins. In addition, the heat transfer from fins was not linearly proportional to the emissivity of the fins. Another study found that the fin efficiency decreases with decreasing thermal conductivity. Finally, the last study concluded that fin effectiveness for annular, isothermal fins, is a function of conductivity, geometry, and emissivity only.

Krishnaprakas [4.7,4.8] noted that for a straight rectangular array of fins with radiation heat transfer only, the optimal design resulted in an array of short, thin fins, implying limitations
of the design. When rectangular fins were attached to an annular heat pipe, it was found that small opening angles resulted in greater fin performance and higher apparent emittances. The opening angle was restricted to a minimum of 40°. Ellison [4.9] studied the gray body shape factor for a rectangular array of fins using a circuit method technique. In comparisons, it was noted that previous studies over or under predicted shape factors. Razelos and Krikiks [4.10] optimized rectangular fins for a given volume and noted that the surface Biot number was a major driving factor. Another key determination was that the radiant heat transfer between the base and fins for a tubular radiator was insignificant and could be ignored. Karlekor and Chao [4.11] solved a similar study with the use of trapezoidal fins instead of rectangular fins. It was found that the optimal number of fins increases with decreasing emissivity. Kumar and Venkateshan [4.12] investigated an annular fin array where the base tube was not isothermal and the gradient varied, depending on the fin heat dissipation. Using this variable base temperature, the effectiveness was calculated and it was found that the effectiveness actually lowered with increasing emissivity. Arslanturk [4.13] studied a radiating longitudinal fin with temperature dependent thermal conductivity and found that the optimal fin dimensions varied by up to 28% from use of a constant thermal conductivity analysis.

Several studies have attempted to optimize the shape of a plate fin to provide greater heat transfer per mass. Keller and Holdredge [4.14] investigated the efficiencies of tapered annular and rectangular fins. Correlations were determined that presented the efficiency as a function of the radius and angle of taper. Naumann [4.15] compared tapered fins to plate fins and found that tapered fins could result in increases in heat transfer per weight of up to 14%. It was also noted that the radiation heat transfer is proportional to the cube root of the thermal conductivity to density ratio. Schnurr et al. [4.16] similarly compared the effects of tapering circular fins along
an annular heat pipe and found that increases in performance could be achieved. A minimum mass design for longitudinal fins extending from a flat base or cylinder was investigated by Chung and Zhang [4.17,4.18]. An optimized shape was determined to be a parabolic surface. Furthermore, the optimal thickness at the base becomes thicker with decreasing emissivity. Using the optimized shape, the optimal spacing was found for an array dependent on the emissivity, heat rate, and conductivity. In another study, Krishnaprakas and Narayana [4.19] found that spine shapes provide the optimal heat transfer per mass. The optimized shape of the fin was found to be independent of mutual irradiation. Wilkins [4.20] found that an optimally shaped curved face rectangular fin could result in a 39% reduction in mass over a rectangular fin. Similarly it was found that a curved face pin fin could result in a 20% reduction in mass over a straight pin fin. Murali et al. [4.21] found that grooves and threads could also be used to increase radiation heat transfer on a mass basis. Grooves were found to be up to 40% more effective on a mass basis while threads were found to be up to 20% more effective on a mass basis.

Another important role in radiation heat transfer is its influence on fin performance. It can be seen in the published literature that thermal radiation accounts for a significant portion of heat transfer in heat sinks operating under natural convection. Rea and West [4.22] noted that thermal radiation heat transfer accounts for approximately 25% of the total heat transfer in a free convection plate fin heat sink. Khor et al. [4.23] investigated the effects of neglecting thermal radiation in heat sink performance design. It was found that thermal radiation accounted for up to 30% of the heat transfer. However, if view factors were not utilized in the addition of thermal radiation in the model, resulting errors in the heat transfer of up to 60% could occur. Abramzon [4.24] developed a closed form solution to solve for the view factors in a heat sink radiating to surroundings. Similarly, it was stated that thermal radiation accounted for over 20% of the total
heat transfer. Yu et al. [4.25] showed that if the thermal radiation, which could account for up to 27% of the total heat transfer, was included in the design of the heat sink, performance increases of up to 12.3% could be achieved while utilizing a more condensed design. Shabany [4.26] developed a simplified expression to determine the view factor in a heat sink, with a maximum error of 11% at the extreme case of fin spacing approaching infinity.

Fractal geometries can lead to large increases in available surface area without large increases in fin size or mass. Furthermore, certain fractal geometries can result in increased surface area with a corresponding reduction of fin mass. In a previous study, Dannelley and Baker [4.27] found that fractal-like fins could improve fin effectiveness as well as fin effectiveness per unit mass for natural convection heat transfer. This was significant as it proved that the increased surface area from fractal-like fins corresponds to an increase in thermal performance. This paper will expand upon previous results by examining the thermal performance of fractal-like fins and their potential to increase thermal performance per unit mass for radiation-only heat transfer. This could be especially useful for spacecraft thermal management as any reduction in mass can result in significant cost savings.

Computational Method

Simulations were performed using a commercially available computational heat transfer software package. The thermal radiation model accounted for both emitted and reflected thermal radiation. The net radiative heat flux is determined as the difference in the thermal radiation emitted and reflected from the thermal radiation absorbed as seen in Equation 4.1.

\[
q'_{\text{r}}' = \epsilon \sigma T^4 + \rho_{\text{r}} q'_{\text{T},i} - \alpha q''_{\text{T},i}
\]  

Kirchhoff’s Law is assumed to hold true, which allows the surface absorptivity to be equal to the surface emissivity. Unless specified as a blackbody, all surfaces are modeled as a
graybody with a user specified emissivity. The value of reflectivity is equal to unity minus absorptivity. The incident thermal radiation on a surface is determined by calculating the view factors utilizing a discrete ray Monte-Carlo approach. Rays are emitted from each surface element such that they are distributed across an enclosing hemisphere by dividing the rays equally along the azimuthal and zenith directions. Each ray is traced from a surface element in a straight line along the defined direction until it either contacts another surface element or reaches the boundary of the computational domain. If a ray contacts another surface element, the irradiation is uniformly distributed across the surface area. The view factor is determined by finding the fraction of the total emitted rays leaving a surface element that contact the desired surface element. After solving for the view factors, the incident thermal radiation on a surface can be defined as the summation of the product of the radiosity and the view factor for all surface elements as shown in Equation 4.2.

\[ q_{T,i} = \sum F_{i,k} \varepsilon \sigma T_{k}^{4} \]  

(4.2)

The governing equation for the system can be found in Equation 4.3.

\[ \nabla^{2}T = 0 \]  

(4.3)

The boundary condition for the fin base is an isothermal temperature as seen in Equation 4.4. The boundary condition for any surface element thermal radiating to the surroundings is the net radiative heat flux as seen in Equation 4.5.

\[ T = T_{b} \quad @S_{b} \]  

(4.4)

\[ k \frac{\partial T}{\partial x_{n}} = \varepsilon \sigma T^{4} + \rho_{T} q'_{T,i} - \alpha q''_{T,i} \quad @S_{rad} \]  

(4.5)

The governing equation is solved with a rectangular computational mesh in the Cartesian coordinate system, with the computational domain constructed of planes orthogonal to the coordinate axes. Using a cell-centered finite volume method, approximations of the governing
equations are obtained and the values of all calculated parameters and physical properties are stored in the center of the mesh cell. Spatial derivatives are approximated with implicit difference operators and are second order accurate.

Validation/Verification

Validation of the computational model was performed with a benchmark solution for a single rectangular fin with an isothermal base and insulated tip radiating to a free space environment [4.28]. A square aluminum fin with a base width of 10.16 cm, base thickness of 0.3175 cm, and surface emissivity of one was used for validation. The base temperature was varied from 200 K to 600 K and the fin efficiency was calculated for each base temperature. A comparison of the computational results and the benchmark solution is presented in Figure 4.1.

A spatial grid refinement was performed for the case with the most complex geometry, which corresponded to the fourth iteration of the Sierpinski carpet with a width to thickness ratio of 64. For grid independence the number of cells was incrementally doubled and the fin effectiveness and fin efficiency were calculated until a change of less than 1% was achieved. A total number of cells of approximately 30,000 was found to be sufficient to achieve the criterion.

Analysis

In this investigation, two fractal geometries were examined to understand how a corresponding increase in fin surface area affects the thermal performance. The two fractal sets that were utilized are the modified Koch snowflake and the Sierpinski carpet. Figure 4.2 displays the zeroth iteration and first three iterations for both fractal sets studied. The zeroth iteration for the modified Koch snowflake is a triangle while the zeroth iteration for the Sierpinski carpet is a square. As iterations are performed for the two fractal sets, the Sierpinski carpet results in a
reduction of fin material while the modified Koch snowflake results in an increase in fin material.

As seen in Figure 4.2, the number and size of squares removed each iteration or number and size of triangles added is dependent on the iteration index. The process for performing iterations on fractal patterns is described in further detail by Devaney [4.29]. While the Sierpinski carpet intuitively appears to have the greatest potential for enhance fin performance on a unit mass basis, due to its ability to increase surface area while reducing mass, both patterns will be evaluated. This will allow for further insight into the behavior of fractal-like fins.

In order to quantify fin performance, fractal-like fins are modeled for the zeroth iteration and first four iterations for all geometric parameters and surface parameters evaluated. An isometric view of the zeroth iteration for a 10.16 cm width and 0.3175 cm thick fin is shown for both fractal patterns in Figure 4.3. In order to easily obtain the surface area of each of the fractal-
like fins at a given iteration, equations were developed dependent on the fin width and thickness. The surface area for a fin utilizing the modified Koch snowflake pattern can be found using Equation 4.6 [4.27] and the surface area of a fin utilizing the Sierpinski carpet can be found using Equation 4.7 [4.27].

\[ A_s(n) = 2 \left( wt + \frac{\sqrt{3}}{4} w^2 \right) + \sum_{i=1}^{n} \left[ \left( \frac{w}{3^n} \right)^2 \left( \frac{\sqrt{3}}{2} \right) + \left( \frac{w}{3^n} \right) t \right] 2^{2n-1} \]  
\[ (4.6) \]

\[ A_s(n) = 2w^2 + 3wt - \sum_{i=1}^{n} 8^{n-1} \left[ 2 \left( \frac{w}{3^n} \right)^2 - 4 \left( \frac{w}{3^n} \right) t \right] \]  
\[ (4.7) \]

The relative change in surface area over the zeroth iteration for each fractal-like fin set can be found in Figure 4.4 for the first six iterations. In contrast, the surface area of the Sierpinski carpet can be significantly modified by varying the width/thickness ratio. For the ratios plotted, the surface area decreases with the first iteration. However, with thicker fins, the surface area is increased beyond the baseline condition with less iteration. For the width/thickness ratios of 16, 32, and 64, the surface area is increased beyond the baseline condition after 3, 4, and 5 iterations respectively. For comparison to the modified Koch snowflake, the Sierpinski carpet pattern results in an increase in surface area of 23% after the fourth iteration for width/thickness ratio of 32.
Similar to the surface area, it is desirable to obtain a direct correlation for the mass of a fin at a given iteration. The mass of a fin utilizing the modified Koch snowflake pattern can be
found using Equation 4.8 [4.27] and the mass of a fin utilizing the Sierpinski carpet pattern can be found using Equation 4.9 [4.27].

\[ m(n) = \left(\frac{\sqrt{3}}{4}\right) w^2 + \sum_{j=1}^{n} \left(\frac{w}{3^n}\right)^2 \left(\frac{\sqrt{3}}{4}\right) 2^{2n-1}\rho t \]  

(4.8)

\[ m(n) = \left[ w^2 - \sum_{j=1}^{n} 8^{n-1} \left(\frac{w}{3^n}\right)^2 \right] \rho t \]  

(4.9)

The mass of the modified Koch snowflake increases with each iteration before converging to a maximum relative change in mass of approximately 40%. The Sierpinski carpet, however, decreases in mass with each iteration and approaches zero mass with infinite iterations. After four iterations, the modified Koch snowflake results in an increase in mass of 38% while the Sierpinski carpet achieves a 38% reduction in mass. As seen by the equations developed for the surface area of a fin utilizing either of the two patterns, the relative change in surface area at a given iteration is a function of fin width/thickness ratio. However, as seen by the equations developed for the mass of a fin utilizing either of the two patterns, the relative change in mass at a given iteration is not a function of width/thickness ratio. The relative change in mass is strictly a function of the fractal pattern chosen.

A baseline case was established that corresponds to an aluminum fin with a fin base width of 10.16 cm, fin base thickness of 0.3175 cm, fin surface emissivity of 1, and a fin base temperature of 350 K. For simplicity, fins were assumed to radiate to surroundings at 0 K. Since radiation interaction between the fin and a base would require additional information such as base material and area, it was neglected. Utilizing the baseline condition, the effect of surface emissivity on the performance of fractal-like fins was studied by varying the surface emissivity from 0.1 to 1. The effect of the width/thickness ratio was evaluated by holding the base width constant and varying the thickness such that the width/thickness ratio was varied from 8 to 64.
This variation represents an increase and decrease in thickness from the baseline condition, which corresponds to a width/thickness ratio of 32. The fin base temperature was varied from 350 K to 500 K to understand the performance with increasing base temperatures. Finally, the effect of the fin material was studied as analysis was performed for additional fin materials consisting of copper, iron, and titanium.

Performance Characteristics

The fin efficiency was calculated using Equation 4.10 and refers to the ratio of heat transfer achieved to the heat transfer that would occur if the entire fin was at the base temperature.

\[
\eta = \frac{Q_r}{A_s \varepsilon_r \sigma T_b^4}
\]  

(4.10)

The fin effectiveness was calculated using Equation 4.11 and refers to the ratio of heat transfer achieved to the heat transfer that would occur if no fin was present.

\[
\varepsilon = \frac{Q_r}{A_b \varepsilon_r \sigma T_b^4}
\]  

(4.11)

Fin effectiveness is the parameter that is typically used when designing fins, although fin efficiency is included for added insight. Finally, the fin effectiveness per unit mass was examined as the reduction in mass is the significant reason for proposing the use of fractal-like fins.

Results

Utilizing the baseline conditions, the performance of the Sierpinski carpet was compared to the modified Koch snowflake to understand the behavior of fractal patterns that add material with iterations versus those that remove material. Results obtained for both the modified Koch snowflake and the Sierpinski carpet are plotted in Figure 4.5.
For the baseline conditions, the modified Koch snowflake results in an initial decrease in fin effectiveness per unit mass with the first iteration. However, the modified Koch snowflake results in positive increases with subsequent iterations and a total increase in effectiveness per unit mass of 1.6% after the fourth iteration. Also, it appears as if the increase in fin effectiveness per unit mass plateaus after the fourth iteration. In contrast, the Sierpinski carpet results in an increase in fin effectiveness per unit mass with each iteration. This results in a total increase of fin effectiveness per unit mass by 24.8% after the fourth iteration. Unlike the modified Koch snowflake, the slope of the fin effectiveness per unit mass at the fourth iteration appears to predict increased performance with a fifth iteration for the Sierpinski carpet. The increased performance of the modified Koch snowflake appears minimal and does not provide any significant improvements, while the increased performance of the Sierpinski carpet is more substantial. Because of this, the Sierpinski carpet will be the only pattern evaluated for the remaining results as its applications are more promising.
The effect of surface emissivity on fin effectiveness for the Sierpinski carpet is plotted in Figure 4.6. The behavior of the fin effectiveness at each iteration is greatly affected by the surface emissivity. As seen for an emissivity of unity, the fin effectiveness monotonically decreases with each iteration. However, once the surface emissivity is decreased to a value of approximately 0.5, the fin effectiveness obtains a minimum value after the second iteration and increases with both the third and fourth iteration. When a surface emissivity of 0.1 is utilized, the fin effectiveness increases largely with the third iteration and results in an increase in fin effectiveness after the fourth iteration, as compared with the zeroth iteration. Because of the previously stated assumptions regarding radiative properties, as the emissivity is reduced, the absorptivity decreases and therefore results in less absorption of inter-surface irradiation due to surfaces having a higher reflectivity. This increase in reflectivity results in a majority of the emitted thermal radiation reaching the surroundings. Consequently, for low values of emissivity the increased surface area from higher order iterations is able to better dissipate heat to the surroundings. The fin effectiveness per unit mass is plotted in Figure 4.7 and is found to increase with each iteration for all emissivities. This behavior shows that while the fin effectiveness may reduce in some cases, such as seen for an emissivity of unity in Figure 4.6, the fin effectiveness per unit mass is still increased. This increase in fin effectiveness per unit mass is a direct result of an 11% reduction of mass with each iteration, allowing the fin to overcome the decreased heat dissipation. The fin efficiency is plotted in Figure 4.8 and gives further insight into the behavior of the fin effectiveness. As seen, the fin efficiencies are increased with decreasing emissivity. In addition, lower emissivity fins do not result in as steep of a decrease in efficiency as higher emissivity fins. These results can be explained similarly to the fin effectiveness as the lower emissivity fins result in a decrease in absorption of inter-surface irradiation.
Figure 4.6. Fin Effectiveness, Variable Emissivity

Figure 4.7. Fin Effectiveness/Mass, Variable Emissivity
The effect of the width/thickness ratio on fin effectiveness for a fin using the Sierpinski carpet is presented in Figure 4.9. The width was held constant and the thickness was varied such that an increasing thickness results in a decreasing width/thickness ratio. For low values of width/thickness ratios, the fin effectiveness is fairly constant with each iteration. As the width/thickness ratio is increased, the fin effectiveness decrease becomes more pronounced with each iteration. This behavior can be attributed to a larger increase in the total surface area as the fin thickness is increased. For comparison, a width/thickness ratio of 16 results in relative increase in surface area of 79% after the fourth iteration and a width/thickness ratio of 64 results in a relative decrease in surface area of 6% after the fourth iteration. Despite decreases in fin effectiveness, increasing width/thickness ratios result in increasing fin effectiveness per unit mass. Although the fin effectiveness is decreased with iteration at each width/thickness ratio, the fin effectiveness per unit mass is increased as seen in Figure 4.10. Despite the fin effectiveness
per unit mass being much smaller for thicker fins, the relative increase in effectiveness per unit mass is much higher. For a width/thickness ratio of 8, the relative change in effectiveness per unit mass is a 46% increase while it is only 17% for a width/thickness ratio of 64. The fin efficiency is shown in Figure 4.11 for each width/thickness ratio. It can be observed that the fin efficiency for the zeroth iteration increases with increasing thickness. However, the fin efficiency for the fourth iteration is reversed, such that the fin efficiency increases with decreasing thickness. This reversal in the fin efficiency can be attributed to a larger increase in inter-surface thermal radiation interaction with increasing iterations for thicker fins. For a width/thickness ratio of 16, the rims due to perforations account for 63.3% of the total fin surface area. In contrast, for a width/thickness ratio of 64, the rim area accounts for only 32.4% of the total fin surface area. Therefore, at higher order iterations, thinner fins result in less surface area for absorption of inter-surface irradiation. This results in higher fin efficiencies at higher order iterations for thinner fins.

![Figure 4.9. Fin Effectiveness, Variable Width/Thickness Ratio](image)

Figure 4.9. Fin Effectiveness, Variable Width/Thickness Ratio
Figure 4.10. Fin Effectiveness/Mass, Variable Width/Thickness Ratio

Figure 4.11. Fin Efficiency, Variable Width/Thickness Ratio
The effect of varying the isothermal base temperature for the Sierpinski carpet was evaluated and the fin effectiveness per unit mass is presented in Figure 4.12. It can be seen that for all base temperatures examined, the fractal-like fins result in increased fin effectiveness per unit mass. With increasing temperature, the fin effectiveness per unit mass is reduced, with higher order iterations resulting in smaller increases when compared with the lower base temperatures. In comparison, for a base temperature of 350 K the increase in fin effectiveness per unit mass is 24.8% while the increase is 18.2% for a base temperature of 500 K. As the base temperature of the fin is increased, the temperature gradient across the profile is increased such that the efficiency is lowered, resulting in lower effectiveness at higher base temperatures. However, it is seen that even for larger base temperatures, a significant increase in fin effectiveness per unit mass can be achieved. The effects of fin material on the fin effectiveness per unit mass of the Sierpinski carpet is presented in Figure 4.13. While copper is often used to achieve higher thermal performance in heat transfer applications, aluminum is found to be a more suitable material when accounting for the mass of the fin. Despite having a thermal conductivity almost two times that of aluminum, copper is significantly outperformed by aluminum as the density of copper is almost three times that of aluminum. Similarly, when compared to titanium and iron, aluminum has a much greater thermal conductivity relative to its density. This was expected as aluminum is typically chosen as the material for fins because of its high thermal conductivity to density ratio.
Figure 4.12. Fin Effectiveness/Mass, Variable $T_b$

Figure 4.13. Fin Effectiveness/Mass, Variable Material
Computational Correlation

A computational correlation of the data was developed to further describe the behavior of fractal-like fins radiating to free space. Employing the Buckingham Pi Theorem as described by Fox and McDonald [4.30], a correlation for the fin effectiveness of the Sierpinski carpet was developed. Six dimensionless parameters were found to quantify the effectiveness and the form of the correlation can be seen in Equation 4.12.

\[
\varepsilon = C \Lambda_n d_1 \Gamma d_2 N d_3 Z d_4 \Phi_n d_5 R_n d_6
\]

The surface area ratio was computed at each iteration as the total surface area divided by the total surface area of the zeroth iteration. The Stark number represents the ratio of radiation to conduction. The face area ratio was computed at each iteration as the total area on the face of the fin divided by the total face area of the zeroth iteration. Finally, the rim area ratio was calculated as unity minus the ratio of the surface area due to rims created by perforations divided by the total surface area. The values for all constants, which can be found in Table 1, were found by minimizing the average difference between the calculated fin effectiveness and the predicted fin effectiveness.

<table>
<thead>
<tr>
<th>Table 4.1</th>
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<td><strong>Computational Fin Effectiveness Correlation Constants</strong></td>
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The correlation described is only applicable for fins with emissivity greater than or equal to 0.8. As emissivity was reduced below 0.8, data points resulted in non-linear behavior compared to the line of perfect correlation. This deviation of the calculated effectiveness from the predicted effectiveness can be explained by the increasing amount of reflection between
surfaces and decrease in absorption of inter-surface irradiation. As seen in Figure 14, for the conditions evaluated, the correlation described results in a maximum error of 3.5% and an average error of 1.6%.

Figure 4.14. Fin Effectiveness Correlation

Conclusions

A computational investigation into the use of fractal-like geometries to enhance radiant fin performance was conducted. Once the computational model was validated and verified, a parametric study was undertaken that examined fin effectiveness and fin efficiency in terms of fractal pattern, width/thickness ratio, emissivity, base temperature, and fin material. Based upon the observed results of the fractal-like fins, the following conclusions may be drawn:

- Fin effectiveness per unit mass is increased by at least 25% for the Sierpinski carpet fins after four iterations.
• Fin effectiveness is reduced less for thicker fins and fin effectiveness per unit mass can be improved by up to 46% for the range of width/thickness ratios tested for the Sierpinski carpet.

• Fin effectiveness per unit mass is found to increase with iterations despite decreasing fin effectiveness for the Sierpinski carpet.

• Aluminum is found to increase fin effectiveness per unit mass on an order of two times that of copper for the Sierpinski carpet.

• The modified Koch snowflake was found inadequate, as it only increased fin effectiveness per unit mass by approximately 1.6%.

• A correlation for the fin effectiveness was developed for Sierpinski carpet fins with emissivity greater than or equal to 0.8 which resulted in a maximum error of 3.5% and an average error of 1.6%.

As demonstrated, the fin effectiveness per mass can be greatly increased through the use of fractal-like fins. This is a result of the removal of material through perforations that result in an increased surface area due to the added perimeter from the cuts. The fin effectiveness per unit mass was found to increase by up to 46% for certain geometric configurations.

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References


CHAPTER 5

CONCLUSION

An analysis of the thermal performance of fractal-like fins for natural convection and thermal radiation was examined through three separate studies to quantify their potential for use. The first investigation into the use of fractal geometries to enhance fin performance was conducted through an experimental study for natural convection. Fractal-like fins were manufactured for the baseline and first three iterations for the Sierpinski carpet and modified Koch snowflake patterns. The conclusions of the experimental study can be summarized below:

- Calculated fin effectiveness was proportional to the total surface area for both fractal patterns tested. After three iterations, fin effectiveness was found to increase by up to 11.4% for the Sierpinski carpet and 44.8% for the modified Koch snowflake.
- Fin effectiveness per unit mass was found to increase with iteration, with increases up to 59% for the Sierpinski carpet and 6% for the modified Koch snowflake.
- Although increased fin effectiveness is achieved with each iteration of the modified Koch snowflake, increased fin effectiveness for the Sierpinski carpet is only available with the third iteration for the 0.0254 m width fins.
- An experimental correlation for fin effectiveness was developed, resulting in a maximum error of 2.36% and 1.4% for the Sierpinski carpet and modified Koch snowflake respectively.

The total surface area was found to be a suitable predictor of fin effectiveness behavior for the first three iterations of both fractal patterns tested. Experimental results predicted that the
fourth iteration of the Sierpinski carpet would produce values of fin effectiveness greater than the baseline for the 0.0508 m width and 0.1016 m width fins.

The second investigation was conducted by computationally modeling fractal-like fins in order to expand upon the experimental results from the first investigation for the Sierpinski carpet. The conclusions of the computational study can be summarized below:

- Fin effectiveness per unit mass was increased after four iterations by at least 37% after four iterations for the geometric parameters and heat fluxes tested.
- Thicker fins resulted in larger increases in fin effectiveness per unit mass, with a fin with width/thickness ratio of 8 resulting in an increase of 303% for the baseline conditions for the fourth iteration.
- Fin effectiveness is tolerant of large heat fluxes as an increase in the heat flux by a factor of 8 only resulted in a 3.7% reduction of the fin effectiveness for the baseline conditions for the fourth iteration.
- For the baseline conditions after four iterations, fin effectiveness was increased by 19% for a fin width of 0.01 m while the fin effectiveness was reduced by 7% for a fin width of 1 m.
- Using the additional computational data points, the previous experimental fin effectiveness correlation was improved such that an average error of 0.84% between calculated and predicted fin effectiveness is achieved.

The computational investigation substantiated the previous experimental investigation and expanded upon the results. Fin effectiveness per unit mass was found to be significantly increased by utilizing a fractal-like fin modeled after the fourth iteration of the Sierpinski carpet.
Finally, the ability of fractal-like fins to enhance heat transfer for thermal radiation was investigated using a computational model. The conclusions of the computational study can be summarized below:

- For the baseline conditions, fin effectiveness per unit mass was increased by a minimum of 25% for the Sierpinski carpet.
- For the baseline conditions, fin effectiveness per unit mass was increased by a larger percentage for thicker fins, with an increase in fin effectiveness per unit mass of 46% for a width/thickness ratio of 8.
- Although fin effectiveness was found to decrease for all conditions tested, fin effectiveness per unit mass was increased with each iteration.
- The optimal fin material was found to be aluminum, with fin effectiveness per unit mass of aluminum fins approximately two times larger than that of copper fins.
- The ability of the modified Koch snowflake to increase radiant heat transfer was found to be negligible.
- A computational correlation for the fin effectiveness was developed for Sierpinski carpet, resulting in a maximum error of 3.5%. The correlation was only applicable for fins with emissivity greater than or equal to 0.8.

The computational investigation of fractal-like fins for radiant heat transfer proved that the loss of fin material due to perforations could result in increased fin effectiveness per unit mass. As seen by the three separate investigations, fractal patterns can be used to increase the performance of extended surfaces. In most cases fractal-like fins were able to increase fin effectiveness per unit mass while in many cases, fin effectiveness was also increased. The results
of this work show the potential for fractal-like fins to be used for applications where decrease in fin mass or size restrictions are desirable.