EFFECTS OF MAGNETIC MATERIAL ON PERFORMANCE OF PERMANENT MAGNET SYNCHRONOUS MACHINES

by

RYAN GRAVES
TIMOTHY HASKEW, COMMITTEE CHAIR
YANG-KI HONG
ANDREW LEMMON
PAUL PUZINASKIS

A THESIS

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ABSTRACT

The effects of permanent magnet material on the performance of PMSM machines is investigated without the use of finite element analysis. A study into magnetic material parameters reveals that the remanent flux and relative recoil permeability of a permanent magnet have the majority of impact on the performance of machines. A rare earth free permanent magnet, MnBiCo, is examined as an alternative material to traditional PMSM magnets. A model for the PMSM is developed paying due diligence to the properties of the magnetic material used in construction of the machine. It is demonstrated that a change in magnetic material will change the torque constant of the motor as well as the inductance of the machine. However, due to most permanent magnet materials having a relative recoil permeability of unity, the change in inductance is negligible. A computer simulation of the PMSM model is developed.

A motion control system is simulated. A commercial motor drive is modeled in this simulation. A commercial PMSM is also modeled in this simulation. Performance tests are conducted on the system in both simulation and hardware experiment. The controllers in the computer simulation of the motor drive are modified in order to accurately represent the behavior of the system in the hardware setup.

A case study of replacing the ferrite permanent magnets in the specified machine is simulated. MnBiCo magnets produce a motor torque constant 3.95 times larger than that of the ferrite magnets. The performance of the two machines is compared and the results are discussed.
It is discovered that the remanent flux of the magnetic material is the sole material property that effects the performance of PMSM machines. The developed simulation will provide motor designers with a quick, simple model for investigating replacement of magnetic material within a PMSM while making no changes to the machine’s dimensions or windings.
DEDICATION

This thesis is dedicated to everyone who has supported me through the process of creating this document. Of particular note this thesis is dedicated to my loving wife and family who allowed me to take the time to complete this thesis.
LIST OF ABBREVIATIONS AND SYMBOLS

PMSM  Permanent Magnet Synchronous Machine

\((BH)_{\text{max}}\)  Maximum Energy Product

\(H\)  Magnetic Field

\(B\)  Magnetic Flux Density

\(B_{\text{sat}}\)  Saturation Flux

\(B_r\)  Remanent Flux

\(H_c\)  Coercive Flux

\(H_{\text{max}}\)  Maximum Magnetic Field Intensity

DC  Direct Current

AC  Alternating Current

\(i(t)\)  Phase Current

\(\gamma\)  Axis Angle

\(\lambda\)  Flux Linkage

\(L\)  Inductance

\(v\)  Phase Voltage

\(\theta_{da}\)  \(d\)-axis Angle in Reference to \(a\)-axis

\(\omega_d\)  Electrical Speed

\(\omega_{\text{mech}}\)  Mechanical Speed

\(R\)  Winding Resistance
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_o )</td>
<td>Permeability of Free Space</td>
</tr>
<tr>
<td>( l )</td>
<td>Length</td>
</tr>
<tr>
<td>( N )</td>
<td>Number of conductor turns</td>
</tr>
<tr>
<td>( p )</td>
<td>Number of Machine Poles</td>
</tr>
<tr>
<td>( r )</td>
<td>Radius of Machine Rotor</td>
</tr>
<tr>
<td>( T )</td>
<td>Torque</td>
</tr>
<tr>
<td>( J )</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>( k_e )</td>
<td>Motor EMF Constant</td>
</tr>
<tr>
<td>( k_t )</td>
<td>Motor Torque Constant</td>
</tr>
<tr>
<td>( \lambda_f )</td>
<td>Motor Flux Linkage Constant</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>Magnetic Flux</td>
</tr>
<tr>
<td>( \Phi_r )</td>
<td>Remanance Flux</td>
</tr>
<tr>
<td>( \Phi_g )</td>
<td>Airgap Flux</td>
</tr>
<tr>
<td>( \Phi_L )</td>
<td>Leakage Flux</td>
</tr>
<tr>
<td>( P )</td>
<td>Permeance</td>
</tr>
<tr>
<td>( P_L )</td>
<td>Leakage Permeance</td>
</tr>
<tr>
<td>( P_{m0} )</td>
<td>Internal Magnet Permeance</td>
</tr>
<tr>
<td>( R_g )</td>
<td>Airgap Reluctance</td>
</tr>
<tr>
<td>( R_{st} )</td>
<td>Stator Tooth Reluctance</td>
</tr>
<tr>
<td>( R_{ry} )</td>
<td>Rotor Steel Reluctance</td>
</tr>
<tr>
<td>( R_{sy} )</td>
<td>Stator Steel Reluctance</td>
</tr>
<tr>
<td>( F )</td>
<td>Magneto Motive Force</td>
</tr>
<tr>
<td>( L_{ph} )</td>
<td>Phase Self-Inductance</td>
</tr>
</tbody>
</table>
\begin{align*}
L_{\text{gap}} & \quad \text{Airgap Self-Inductance Component} \\
L_{\text{slot}} & \quad \text{Slot Self-Inductance Component} \\
L_{\text{end}} & \quad \text{End Turn Self-Inductance Component} \\
M_{\text{ph}} & \quad \text{Phase Mutual-Inductance} \\
M_{\text{gap}} & \quad \text{Airgap Mutual-Inductance Component} \\
M_{\text{slot}} & \quad \text{Slot Mutual-Inductance Component} \\
M_{\text{end}} & \quad \text{End Turn Mutual-Inductance Component} \\
g' & \quad \text{Equivalent Airgap} \\
K_c & \quad \text{Carter’s Coefficient} 
\end{align*}
ACKNOWLEDGMENTS

I would like to take this opportunity to communicate how thankful I am to my colleagues, friends, mentors, and faculty members. Without their support, this thesis would not be possible. The chairman of this thesis is Dr. Tim Haskew, professor and head of the Department of Electrical and Computer Engineering at The University of Alabama. Dr. Haskew has been my closest mentor throughout the duration of this research project. Whenever I hit a seemingly dead end in my research, Dr. Haskew provided guidance and words of wisdom stemming from his prolific research expertise. I would also like to thank Dr. Yang-Ki Hong of the Devices and Materials research group at The University of Alabama. Dr. Hong was the Principle Investigator for the ARPA-E project that sponsored the research conducted in this thesis. Without sponsorship from this program, this thesis would not have been possible.

I would like to express my appreciation to all members, faculty and students alike, of the Electromechanical Systems Laboratory at the University. The engineering expertise in this collective is astounding. The instrumentation and equipment provided in this laboratory made simple work of all hardware experiments conducted in this thesis. This thesis would not have been possible without this Laboratory.

Finally, I would like to thank my friends and colleagues. Their encouragement has seen me through to the completion of this research project and thesis.
CONTENTS

ABSTRACT................................................................................................................................. ii

DEDICATION............................................................................................................................ iv

LIST OF ABBREVIATIONS AND SYMBOLS ............................................................................... v

ACKNOWLEDGMENTS .............................................................................................................. viii

CONTENTS............................................................................................................................... ix

LIST OF TABLES ..................................................................................................................... xii

LIST OF FIGURES ................................................................................................................... xiii

CHAPTER 1: INTRODUCTION ...................................................................................................... 1
  1.1 Rare Earth Materials ........................................................................................................ 1
  1.2 Rare Earth Permanent Magnets ........................................................................................ 1
  1.3 Market Conditions .......................................................................................................... 2
  1.4 ARPA-E REACT ............................................................................................................. 3
  1.5 Literature Review .......................................................................................................... 5
  1.6 Thesis Organization ....................................................................................................... 6

CHAPTER 2: PERMANENT MAGNETS ...................................................................................... 8
  2.1 B-H Curves .................................................................................................................... 9
  2.2 Common Magnetic Materials in Motors and MnBiCo .................................................... 10
  2.3 Performance at Elevated Temperatures ......................................................................... 13
6.2.2 Armature Inductance ................................................................. 53
6.2.3 Motor Constant Measurements .................................................. 55
6.2.4 Equivalent Mechanical Parameters ............................................ 57

CHAPTER 7: MODELING THE MOTOR DRIVE ........................................... 61
7.1 Introduction to Motor Drives .......................................................... 61
7.2 Power Electronics ........................................................................ 62
7.3 Hardware Motor Drive .................................................................. 65
7.4 Motor Drive Hardware .................................................................. 68
7.5 Motor Drive Controllers ............................................................... 69

CHAPTER 8: SYSTEM LEVEL SIMULATION AND RESULTS ......................... 75

CHAPTER 9: SYSTEM LEVEL SIMULATION OF MOTOR WITH NEW MATERIAL ...... 84

CHAPTER 10: CONCLUSION .................................................................. 92
10.1 Limitations and Future Work ......................................................... 95
10.2 Significance .................................................................................. 95

REFERENCES ....................................................................................... 96
LIST OF TABLES

Table 2.1: Primary Intrinsic Properties of Permanent Magnets............................................. 11

Table 6.1: Four Terminal Resistance Test Results ................................................................. 53
LIST OF FIGURES

Figure 1.1: Rare Earth Material Production by Country [5] .............................................................. 3

Figure 2.1: B-H Curve ......................................................................................................................... 10

Figure 2.2: Demagnetization Curves of Common Permanent Magnets ............................................. 11

Figure 2.3: Demagnetization Curves of Sm2Co17 and MnBiCo ....................................................... 12

Figure 2.4: Normal Demagnetization Curves of SmCo and MnBiCo at 200°C ............................... 14

Figure 3.1: abc and dq Axes .............................................................................................................. 17

Figure 3.2: αβ and dq-axes ................................................................................................................. 19

Figure 4.1: Image Illustrating the Flux Path of a Radial Machine .................................................. 26

Figure 4.2: Magnetic Equivalent Circuit of Radial Flux PMSM .................................................... 28

Figure 4.3: Simplified Magnetic Equivalent Circuit of Radial PMSM ........................................... 29

Figure 4.4: Simplified Magnetic Equivalent Circuit of Radial PMSM ........................................... 30

Figure 5.1: Flux Distribution in the Gap ............................................................................................ 38

Figure 5.2: Phasor Diagram of a Three-Phase Short Circuit ............................................................. 42

Figure 6.1: Simulink Model of Permanent Magnet Synchronous Machine ..................................... 47

Figure 6.2: Electrical Model .............................................................................................................. 47

Figure 6.3: dq-Axis Currents ............................................................................................................. 48

Figure 6.4 q-Axis Current Modeled in SIMULINK ......................................................................... 49

Figure 6.5: d-Axis Current Modeled in SIMULINK ......................................................................... 49

Figure 6.6: Mechanical Equations Modeled in SIMULINK ............................................................. 50
Figure 6.7: Available Measurements and Feedback Signals in Model........................................51
Figure 6.8: Four-Terminal Resistance Measurement..........................................................53
Figure 6.9: Jones Bridge for Inductance Measurement [22]...............................................54
Figure 6.10: EMF Test at Various Speeds........................................................................56
Figure 6.11: Nominal Shaft Torque Under Steady State Conditions.................................58
Figure 6.12: System Spin-Down Test................................................................................60
Figure 7.1: Basic Half-Bridge Inverter .............................................................................62
Figure 7.2: Basic Full-Bridge Inverter................................................................................63
Figure 7.3: Basic Three-Phase Inverter ............................................................................63
Figure 7.4: Three-Phase Inverter with DC Link Capacitor...............................................64
Figure 7.5: Motor Drive Controller Loops .......................................................................66
Figure 7.6: Programming the Motor Drive Control Loops...............................................66
Figure 7.7: Programming the Velocity Loop .....................................................................68
Figure 7.8: Motor Drive Model .......................................................................................69
Figure 7.9: Simulated Motion Control System.................................................................70
Figure 7.10: Velocity Loop Block ....................................................................................71
Figure 7.11: Commutation Loop Block ............................................................................72
Figure 7.12: Current Loop Block......................................................................................73
Figure 7.13: PWM Generator Block ................................................................................74
Figure 7.14: PWM Generation Block Functionality...........................................................74
Figure 8.3: Optimized Controller System Performance....................................................76
Figure 8.4: Hardware Test Configuration with SE1128 Coupled to Dynamometer........77
Figure 8.5: Results of Hardware Test ..............................................................................79
Figure 8.6: Results of Optimized Simulation with Hardware Test Results........................................ 80
Figure 8.7: Results of Retuned Simulation with Hardware Results for Comparison ................. 81
Figure 8.8: Results of 1000 rpm Test with 0 to 20 Nm Step Change in Load Torque .............. 82
Figure 8.9: Results of Test at 20 Nm 500 rpm to 1750 rpm ......................................................... 83
Figure 9.1: Comparison of MnBiCo to Ferrite Magnet Motor Performance (Shaft Speed)....... 88
Figure 9.2: Comparison of MnBiCo to Ferrite Magnet Motor Performance (Current).......... 89
Figure 9.3: Comparison of MnBiCo to Ferrite Magnet Motor Performance (Shaft Speed)....... 90
Figure 9.4: Comparison of MnBiCo to Ferrite Magnet Motor Performance (Current)......... 91
CHAPTER 1: INTRODUCTION

1.1 Rare Earth Materials

Rare earth materials are elements found in the lanthanide group of the periodic table. They consist of lanthanum, cerium, praseodymium, neodymium, promethium, samarium, europium, gadolinium, terbium, dysprosium, holmium, erbium, thulium, ytterbium, lutetium, and yttrium. The term “rare” earth materials was coined early in the years of employing these materials. In actuality, this group of materials is not rare; in fact, even the least abundant of the rare earth materials, lutetium and thulium, are almost 200 times more abundant in the Earth’s crust than gold. While rare earth materials are abundant, they appear in concentrations only in a handful of locations around the world. This leads to very few supplier sites, as the economics of extraction become an issue [1]. Rare earth materials have diverse nuclear, metallurgical, chemical, catalytic, electrical, optical, and magnetic properties that make the materials useful in emerging technologies. Examples of rare earth material in industry and commercial applications include: europium as a red phosphor in LCD screens, fiber optic cables, and lasers; cerium as a polishing agent for optics; and lanthanum in nickel metal hydride batteries [2]. Of particular note in the power industry are samarium, neodymium, and dysprosium. This particular set of rare earth elements are employed in permanent magnets.

1.2 Rare Earth Permanent Magnets

Rare earth permanent magnets are commonly used in electric components such as motors, generators, and sensors. Applications for such motors and generators range from the incredibly large, such as wind turbines, to the incredibly small, such as the vibrator motor in cell phones. Rare earth permanent magnets are used in these machines as a result of their relatively large maximum energy product, \((BH)_{\text{max}}\). Samarium-cobalt magnets can have maximum energy
products upwards of 33 MGOe, and neodymium-iron-born magnets can have maximum energy products upwards of 52 MGOe [3] [4]. These relatively high maximum energy products make rare earth permanent magnets critical components of motors and generators where high energy density is a principle design goal.

1.3 Market Conditions

According to a report published by the Congressional Research Service, the United States was once self-reliant with respect to the production and consumption of rare earth materials [5]. Currently however, the United States is nearly 100% reliant on imported rare earth materials. The reason behind the shift to consuming imported rare earth materials is the lower cost of manufacturing overseas. The majority of rare earth material imports currently are from China [5]. In 2011, China produced 105,000 metric tons of rare earth material; this makes up 96.8% of all rare earth material production in the world. India produced the next highest amount of rare earth material in 2011 at a mere 2,800 metric tons, or 2.6%. The United States produced 0 metric tons of rare earth material in 2011. Recently, Molycorp Corporation reopened the Mountain Pass rare earth mine in California; this mine produced the bulk of rare earth materials used globally during the 1960’s through the 1980’s but was closed in 2002. Molycorp is currently on pace to produce 15,000 metric tons of rare earth material this year [5]. Figure 1.1 illustrates the United States’ rare earth material imports by country as of 2011. It is clear from the figure that the majority of the United States’ rare earth material imports come from China. The figure also illustrates the dwindling rare earth material reserves of the United States as compared to other nations.
The United States legislature has committed to the research and development of substitutes and alternatives to rare earths. This has prompted the Department of Energy to sponsor the ARPA-E REACT program.

1.4 ARPA-E REACT

The Department of Energy’s Advanced Research Projects Agency – Energy (ARPA-E) Rare Earth Alternatives in Critical Technologies (REACT) program sponsors University and National Laboratory research groups. The University of Alabama is one of such research programs participating in ARPA-E REACT. A research group at the University is currently studying the use of MnBiCo magnets for use in electric vehicle motors and wind turbine generators.
The overall aim of the ARPA-E REACT program is to reduce the United States’ rare-earth material imports from overseas. Successful achievement of this objective would have several important implications. First, if the materials and magnets used in consumer products are mined and produced in America, the money spent on these materials and magnets stays in America. If a viable magnet is created through this program and gains traction in the market, manufacturers may produce these magnets domestically, resulting in job creation. The United States currently spends close to one billion dollars a day on petroleum imports [5]. Leveraging new magnetic materials in the manufacture of electric vehicles could result in a more affordable product for consumers and a reduction of dependence on petroleum imports. Next, if a resulting magnet is successfully deployed in electric vehicles and wind generators, this could result in a reduction of greenhouse gases released into the atmosphere. Reducing the United States’ dependence on foreign nations’ raw materials enhances national security by isolating the nation from areas of economic, political, and social unrest.

One challenge for this optimistic prospect is the existence of a knowledge gap between the understanding of magnetic materials and the understanding of machines. Particularly, little effort has been applied to understanding how changing the magnetic material in a permanent magnet synchronous machine affects its performance. Finite element analysis is becoming an increasingly popular strategy in motor design and this technique is capable of determining these effects. However, there exists no published, basic understanding of how and why changing the magnetic material changes performance. The goal of this thesis is to address this knowledge gap; to provide a basic set of governing equations describing this relationship; and to produce a simulation describing the effect of magnetic material on motor performance without the use of finite element analysis.
1.5 Literature Review

There currently exists a gap in the literature between the materials knowledge and the machines knowledge. When comparisons and studies are presented, they are strictly presented on the performance of machines with differing materials. This thesis aims to bridge the gap between the materials world and the machines world and aims to elucidate how and why machine performance is impacted under the influence of differing magnetic materials.

Applications for a rare earth material replacement include electric vehicles, hybrid-electric vehicles, and wind turbines. Rare earth materials, neodymium-iron-boron in particular, have a relatively high energy density that makes them suitable for vehicle application. Rare earth materials also have higher energy products than ferrite materials. Ironically, there has been a recent shift back to using ferrite materials in electric machines due to the ever increasing cost of rare earth materials [6]. This demonstrates the willingness of motor designers to consider using rare earth alternatives if economically profitable.

Many comparisons of motors with differing magnetic materials have been made. However, these comparisons offer no insight into how the differing magnetic material effects the performance. Only reported is to what degree the performance is impacted. Often, a 2-D or 3-D finite element analysis is given as justification for the resulting performance differences. In these studies, no attempt at developing expressions describing the physical parameters of the machine has been made [6] [7] [8].

The inductance of permanent magnet synchronous machines has been exhaustively studied in terms of self, mutual, and leakage inductance. However, there is a disconnect between the study of inductance of permanent magnet synchronous machines and the study of magnetic materials. Often times, the inductance is studied in the stationary abc-reference frame with
respect to load angle, but these papers fail to take into account the magnetic properties of the material [9]. Some evaluations of inductance take the B-H curves of stator steel into account, but fail to mention the B-H curve of the magnetic material on the rotor of the machine [10].

Some attention has been given to solving the magnetic equivalent circuit within the permanent magnet synchronous machine. In particular, an advanced mesh analysis method of solving the magnetic circuit is demonstrated and even compared to results from finite element analysis [11]. This paper presents a successful mesh analysis of the magnetic equivalent circuit with good accuracy and reduced computational time. No mention of machine inductance is given, however. Another paper, [12], demonstrates a unipolar permanent magnet synchronous machine model will be the basis of the simple magnetic equivalent circuit demonstrated in Chapter 4. Rasmussen also proposes a method of magnetic equivalent circuit analysis that results in an accurate motor model [12]. However, the number of iterative calculations involved encroaches upon finite element analysis.

1.6 Thesis Organization

This thesis attempts to bridge the aforementioned gap in the literature. Also, a model of the permanent magnet synchronous machine requiring the entry of magnetic material properties is developed. This model will assist motor designers with the seemingly simple task of replacing just the magnetic material in a motor, while making no changes to the stator, windings, or rotor. This thesis is organized as follows. Chapter 2 introduces common properties of rare earth permanent magnets in an attempt to understand the properties that will effect motor performance. In Chapter 3, a model of the permanent magnet synchronous machine is developed. In Chapter 4, a simplified magnetic equivalent circuit is analyzed in order to determine how the properties of a permanent magnet effect the model of the permanent magnet synchronous machine. In
Chapter 5, the inductances of the machine are analyzed with respect to magnetic parameters. In Chapter 6, a hardware machine is selected as a test platform for hardware verification of the developed model. The hardware machine is tested for the parameters necessary to complete a computer simulation of the model. In Chapter 7, a hardware motor drive is selected and modeled in the computer simulation. Both the motor drive hardware and controllers are modeled. In Chapter 8, hardware experiments are performed on the selected motor and compared to the results of comparable system simulations. In Chapter 9, a new model for the permanent magnet synchronous machine with respect to magnetic parameters is presented. The hardware motor is evaluated using predicted material parameters, and the results are compared to the hardware experiments.
CHAPTER 2: PERMANENT MAGNETS

A permanent magnet is a magnet constructed of hard magnetic materials. A hard magnetic material is a material that requires a very large field in order to become saturated. Permanent magnets can be thought of as energy storage elements. An amount of energy is placed into the magnet upon magnetization of material. This energy then remains in the magnet indefinitely. Permanent magnets are often used in motors and generators in the power conversion industry.

Magnetic steels were the first material to be considered a permanent magnet. Magnetic steels quickly fell out use with the development of Alnico. Alnico followed magnetic steels in the development timeline of permanent magnet materials. Alnico contains iron (Fe), cobalt (Co) and nickel (Ni) along with aluminum (Al). Rare earth materials began being used in permanent magnets in the 1960’s. The evolution of rare earth magnets progressed quickly with the availability of rare earth materials such as neodymium and samarium, as discussed in Chapter 1. Common materials used in the power conversion industry are samarium-cobalt (SmCo) and neodymium-iron-boron (NdFeB). NdFeB is thought to be the most powerful of the rare earth permanent magnets, while SmCo is prized for its performance at temperatures up to 200°C. Due to the supply constraints, economics, and politics discussed in Chapter 1, recent permanent magnet research has been focused on the creation of rare earth-free permanent magnets with performance similar to that of NdFeB and SmCo. Researchers at The University of Alabama have successfully simulated and proven a new magnetic material: manganese-bismuth-cobalt (MnBiCo). MnBiCo has a similar energy product to that of NdFeB and similar high temperature performance to that of SmCo.
2.1 B-H Curves

B-H curves are employed to describe the magnetization and demagnetization process of permanent magnets. B-H curves also define several useful values and parameters of magnetic materials. A standard B-H curve with labeled points of interest is shown below in Figure 2.1. The x-axis represents the external field applied to a material. The y-axis represents the flux density of the magnetic material. A non-magnetized material sample with no field applied is said to be at the origin on the B-H curve. In order to magnetize the material, a positive field, $H$, is applied to the material. As $H$ rises, so does the flux of the material. This trend continues to a point of saturation, $B_{sat}$. Upon saturating the material, the external field can be removed, and the flux density of the magnet drops to $B_r$, this parameter is said to be the remanant flux of the material. If a negative field is applied to the material, the flux density follows a downward trend. The point at which the flux density crosses zero is called the coercive flux, $H_c$. This represents the coercivity of the magnet and is the point at which a magnet can become demagnetized. If the magnitude of the applied field continues to increase in the negative direction, the flux density will continue to become more negative until a point of saturation $-B_{sat}$. Upon releasing the negative field on the material, the material will return to a negative remanent flux point, $-B_r$. If a positive field is applied to the material, the material flux density will increase to a point of positive saturation again.
2.2 Common Magnetic Materials in Motors and MnBiCo

In the world of motor and generator design, machines are typically operated in the second quadrant of the B-H curve. This is where a “negative” field is applied to the magnetic material. This can be thought of as the windings of a motor causing a field in the airgap that results in the rotation of the rotor. The curves in just this quadrant are referred to as the demagnetization curve of a material. The demagnetization curves of NdFeB, Sm$_2$Co$_{17}$, and MnBiCo are shown below in Figure 2.2. Values of interest for each curve are provided in Table 2.1.
Figure 2.2: Demagnetization Curves of Common Permanent Magnets

<table>
<thead>
<tr>
<th></th>
<th>NdFeB</th>
<th>Sm2Co17</th>
<th>MnBiCo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic $B_r$ (T)</td>
<td>1.30</td>
<td>0.90</td>
<td>0.83</td>
</tr>
<tr>
<td>Intrinsic $H_c$ (MA/m)</td>
<td>1.19</td>
<td>1.99</td>
<td>2.39</td>
</tr>
<tr>
<td>Intrinsic $BH_{\text{max}}$ (MJ/m$^3$)</td>
<td>1.34</td>
<td>1.20</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Table 2.1: Primary Intrinsic Properties of Permanent Magnets

Note from Table 2.1, that the remanence of MnBiCo is 64% of that for NdFeB and 92% of that for Sm$_2$Co$_{17}$. However, the normal demagnetization curves are the most significant for actual motor design. The analysis will be constrained to the comparison of Sm$_2$Co$_{17}$ and MnBiCo. Based on SI units, expressions for the normal demagnetization curves are presented in equations 1 and 2 below. The subscript 1 denotes Sm$_2$Co$_{17}$ and the subscript 2 denotes MnBiCo.

$$B_1 = 0.9 + \mu_0 H_1$$
(2.1)

$$B_2 = 0.83 + \mu_0 H_2$$
(2.2)
Figure 2.3: Demagnetization Curves of Sm2Co17 and MnBiCo

Based on the general forms of equations 2.1 and 2.2, the maximum normal energy product can be found to be that expressed in equation 2.3, and the magnetic field intensity at which the maximum normal energy product occurs is expressed in equation 2.4.

\[
(BH)_{\text{max}} = \frac{-B_r^2}{4\mu_0} \tag{2.3}
\]

\[
H_{\text{max}} = \frac{-B_r}{2\mu_0} \tag{2.4}
\]

Evaluation of (2.3) and (2.4) for Sm2Co17 yields \((BH)_{\text{max}}^{\text{norm}} = 0.161 \text{ MJ/m}^3\) at a magnetic field intensity of \(H_{\text{max}}^{\text{norm}} = -0.358 \text{ MA/m}\). The permeance line indicated in Figure 2.3 intersects the Sm2Co17 normal demagnetization curve at this point. The permeance line is dominated by the air-gap in the motor, and normal operation doesn’t approach this point. However, consideration of this point results in conservative evaluation for the use of MnBiCo. The points of intersection of the permeance line and the normal demagnetization curves for both materials, \(P_1\) and \(P_2\), are indicated below in equations 5 and 6 [14].
Thus, with identical motor designs, the air-gap flux density with MnBiCo will be 92.2% of what it would be with Sm$_2$Co$_{17}$.

2.3 Performance at Elevated Temperatures

One key issue in the performance of permanent magnet synchronous motors, particularly in automotive applications, is the behavior of the magnetic materials at elevated temperature. Care must be taken not to encroach on the Curie temperature of the material. While NdFeB magnets will clearly provide the best motor performance at 25 °C, when the temperature reaches approximately 100 °C, Sm$_2$Co$_{17}$ becomes the better choice. This assessment is based on data from Electron Energy Corporation (EEC) [3] [4]. Since temperatures approaching 200°C are expected in the motor applications intended for this comparison, it is instructive to consider the performance of these materials in this temperature range. For this reason, the comparison will be between Sm$_2$Co$_{17}$ and MnBiCo.

EEC data for Sm$_2$Co$_{17}$ at 200°C indicates, from the normal demagnetization curve, $B_r = 1.07 \, T$ and $H_C = 757.58 \, \text{kA/m}$. At this temperature, the normal demagnetization curve remains linear. The thermal characteristics of MnBiCo can be approximated based upon results presented in [15], which indicate that in the range from 300 K to 500 K, the remanence of MnBiCo decreases linearly by 22.2%. Thus, a decrease in remanence of 0.111 %/°C is expected. Figure 2.4 shows the normal demagnetization curves for these materials at 200°C and the same permeance line used above. The point of intersection with the Sm$_2$Co$_{17}$ curve yields a flux density of 0.53 T, and the point of intersection with the MnBiCo curve yields a flux density of
0.33 T. This indicates the MnBiCo magnet is more susceptible to increased temperature than the Sm$_2$Co$_{17}$ magnet.

![Figure 2.4: Normal Demagnetization Curves of SmCo and MnBiCo at 200°C](image)
CHAPTER 3: PERMANENT MAGNET SYNCHRONOUS MACHINE MODELING

This chapter will discuss the modeling and governing equations of a non-salient pole, sinusoidally wound, radial-flux permanent magnet synchronous machine. A synchronous machine is a machine that employs a constant rotor field [16]. Establishing this constant rotor field can be accomplished through a field winding with a DC current, or in the case of the permanent magnet synchronous machine, a permanent magnet located on the rotor. The constant rotor field synchronizes the rotation of the rotor, and thus the shaft, with the rotating magnetic field induced by current in the three phase, sinusoidally wound stator windings [16]. So it can be said that the rotor rotates synchronously with the frequency of the applied voltage at the terminals of the machine, hence the name synchronous machine. The model derived in this chapter describes a radial-flux machine. A radial-flux machine has a radial flux path. Some machines have an axial flux path and are referred to as axial-flux machines [17]. Saliency describes any change of inductance with a varying rotor position. The machines analyzed in this paper will be non-salient pole machines. Surface mount permanent magnet synchronous machines are an example of non-salient pole machines [18]. The shape of the rotor and positioning of the magnets determines the saliency of a machine. Figure 3.1a illustrates a two-pole, non-salient, sinusoidally wound, radial-flux permanent magnet synchronous machine. This machine will be the basis of the model derived in this chapter.

3.1 A Case for DQ

Analyzing a three-phase motor in the abc-winding reference frame allows for excellent analysis under steady-state conditions. When attempting to analyze a motor in a transient state utilizing the abc-winding reference frame, the task becomes increasingly difficult. As described in [18] [17], the terminal voltage equations, when developed in the abc-winding reference frame,
include a time-derivative of the flux linkage equations which are dependent upon rotor position.

To simplify motor transient analysis, an alternate reference frame called the \(dq\)-winding reference frame is used in the literature. The \(dq\) reference frame is a rotating, two-phase reference frame based on the Park’s transformation [18]. When examining Park’s transform for the first time, it is often easier to start with Clarke’s transform [18]. Clarke’s transform allows us to transform a three-phase system to a two-phase system.

\[
\begin{bmatrix}
i_d(t) \\
i_q(t)
\end{bmatrix} = \begin{bmatrix}
1 & \cos(\gamma) & \cos(2\gamma) \\
0 & \sin(\gamma) & \sin(2\gamma)
\end{bmatrix} \begin{bmatrix}
i_a(t) \\
i_b(t) \\
i_c(t)
\end{bmatrix}
\] (3.1)

\[
\gamma = \frac{2\pi}{3}
\] (3.2)

The \(\alpha\)-axis is aligned with the \(a\)-axis. The \(\beta\)-axis is orthogonal to the \(\alpha\)-axis. In space vector form:

\[
i_{sa\beta}(t) = i_a(t) + ji_\beta(t)
\] (3.3)

To complete the Park’s transform, the rotating component must be applied.

\[
\begin{bmatrix}
i_d(t) \\
i_q(t)
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
i_a(t) \\
i_\beta(t)
\end{bmatrix}
\] (3.4)

\[
\theta = \omega t
\] (3.5)

It is possible to convert straight from the \(abc\)-winding reference frame to the \(dq\)-winding reference frame using the following equations.

\[
\begin{bmatrix}
i_d(t) \\
i_q(t)
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
\cos(\theta_{da}) & \cos(\theta_{da} - \frac{2\pi}{3}) & \cos(\theta_{da} - \frac{4\pi}{3}) \\
-\sin(\theta_{da}) & -\sin(\theta_{da} - \frac{2\pi}{3}) & -\sin(\theta_{da} - \frac{4\pi}{3})
\end{bmatrix} \begin{bmatrix}
i_a(t) \\
i_b(t) \\
i_c(t)
\end{bmatrix}
\] (3.6)
\( \theta_{da} \) is the angle between the magnetic axis of the rotor and the \( a \)-phase considering the \( d \)-axis is always aligned with the magnetic axis of the rotor.

![Diagram](image-url)

**Figure 3.1: abc and dq Axes [18]**

An argument can be made that using the computing power of today, there is no need for the \( dq \)-transform. This is incorrect. Not only does the \( dq \)-winding reference frame allow for simplified governing equations, it also allows for the torque and field of a motor to be controlled independently of each other [18]. This will be examined later in the chapter. So, with knowledge of the \( dq \)-transform, the model of the permanent magnet synchronous machine can be developed.

### 3.2 Flux linkage Equations

It is well-known that flux linkage is the product of a coil’s inductance and the current flowing through the coil; this relationship is described by the following equation, where \( \lambda \) is flux linkage, \( L \) is the inductance of a coil, and \( i \) is the current in the coil.

\[
\lambda = Li
\]  

(3.7)
The stator flux linkage equations of a permanent magnet synchronous machine are developed in the $dq$ reference frame and described as:

$$\lambda_{sd} = L_s i_{sd} + \lambda_{fd}$$  \hspace{1cm} (3.8) \\
$$\lambda_{sq} = L_s i_{sq}$$  \hspace{1cm} (3.9)

where $L_s$ is the stator inductance, $\lambda_{fd}$ is the flux linkage due to the magnets on the rotor, $i_{sd}$ is the current in the stator $d$-axis winding, and $i_{sq}$ is the current in the stator $q$-axis winding [18]. Chapter 5 will describe machine inductances in further detail. The stator $d$-axis is always aligned with the magnetic axis of the rotor. This gives the additional $\lambda_{fd}$ term in the stator $d$-axis flux linkage equation.

3.3 Stator Voltage Equations

![Diagram of $\alpha\beta$ and $dq$-axes](18)
The stator winding voltage equations are developed as follows. First, Clarke’s transform is applied to the three-phase, sinusoidally wound abc-windings. This produces the \( \alpha\beta \)-windings. The \( \alpha\beta \)-windings are stationary windings separated by 90\(^\circ\) electrical with the \( \alpha \)-winding aligned with the stator \( a \)-axis. Through the use of Ohm’s law and a simplified version of Faraday’s law, the stator \( \alpha\beta \)-winding voltages are derived as follows:

\[
\begin{align*}
v_{s\alpha} &= R_s i_{s\alpha} + \frac{d}{dt} \lambda_{s\alpha} \\
v_{s\beta} &= R_s i_{s\beta} + \frac{d}{dt} \lambda_{s\beta}
\end{align*}
\]

(3.10) (3.11)

Applying the product rule to Faraday’s Law yields (3.12). A simplified version of Faraday’s law neglects any change of inductance in time (3.13).

\[
\begin{align*}
v &= L \frac{d}{dt} i + i \frac{dL}{dt} \\
v &= L \frac{d}{dt} i
\end{align*}
\]

(3.12) (3.13)

The stator \( \alpha\beta \)-winding voltage equations can be combined into space vector form by multiplying the \( \beta \)-winding by \( j \) and summing the two.

\[
\begin{align*}
\overrightarrow{v_{s\alpha\beta}} &= v_{s\alpha} + j v_{s\beta} \\
\overrightarrow{v_{s\alpha\beta}} &= R_s \overrightarrow{i_{s\alpha\beta}} + \frac{d}{dt} \overrightarrow{\lambda_{s\alpha\beta}}
\end{align*}
\]

(3.14) (3.15)

The \( \alpha\beta \)-axes quantities can be related to the \( dq \)-axes quantities simply by rotating the component \( te^{j\theta_{da}} \), where \( \theta_{da} \) is the angle between the \( a \)-phase and the \( d \)-axis.
$$\overline{v_{s,\alpha\beta}} = \overline{v_{s,dq}}e^{j\theta_{da}}$$  \hfill (3.16)$$

$$\overline{i_{s,\alpha\beta}} = \overline{i_{s,dq}}e^{j\theta_{da}}$$  \hfill (3.17)$$

$$\overline{\lambda_{s,\alpha\beta}} = \overline{\lambda_{s,dq}}e^{j\theta_{da}}$$  \hfill (3.18)$$

Substituting the previous three equations into the \(\alpha\beta\)-winding voltage equations yields the following expression.

$$\overline{v_{s,dq}}e^{j\theta_{da}} = R_s\overline{i_{s,dq}}e^{j\theta_{da}} + \frac{d}{dt}(\overline{\lambda_{s,dq}}e^{j\theta_{da}})$$  \hfill (3.19)$$

Applying the product rule to the time derivative term yields the following expression.

$$\overline{v_{s,dq}}e^{j\theta_{da}} = R_s\overline{i_{s,dq}}e^{j\theta_{da}} + \frac{d}{dt}\overline{\lambda_{s,dq}}e^{j\theta_{da}} + j\frac{d\theta_{da}}{dt}\overline{\lambda_{s,dq}}e^{j\theta_{da}}$$  \hfill (3.20)$$

Note that \(\frac{d\theta_{da}}{dt} = \omega_d\), so it can be said that the dq-winding voltage equations are as follows:

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = R_s \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix} + \omega_d \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix}$$  \hfill (3.21)$$

It was stated earlier that the flux linkages are described by the following equation.

$$\begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix} = L_s \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \lambda_{fd} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$  \hfill (3.22)$$

By substituting the flux linkage equation into the dq-winding voltage equation the governing equations of stator dq-winding voltages are found.

$$v_{sd} = R_s i_{sd} + L_s \frac{d}{dt} i_{sd} - \omega_d L_s i_{sq}$$  \hfill (3.23)$$

$$v_{sq} = R_s i_{sq} + L_s \frac{d}{dt} i_{sq} - \omega_d (L_s i_{sd} + \lambda_{fd})$$  \hfill (3.24)$$
The flux linkage and winding voltage equations for a permanent magnet synchronous machine have now been derived. The goal of a motor is to transform electrical energy into mechanical energy, so the next step is to derive the equations relating electrical terms to mechanical terms.

3.4 Electromagnetic Torque Equations

The developed electromagnetic torque on the rotor is the link between the electrical and mechanical models of a permanent magnet synchronous machine. Just as was done for the flux linkage and stator voltage equations, the expression for the developed torque is derived in terms of the $dq$-axes.

The flux densities along each axis are described as follows where $B_{sd}$ is the flux density along the stator $d$-axis, $B_{sq}$ is the flux density along the stator $q$-axis, $\mu_0$ is the permeability of free space, $l_g$ is the length of the airgap, $N_s$ is the number of conductor turns of the stator winding, $M$ is the phase mutual inductance, $L_s$ is the phase self-inductance, and $p$ is the number of machine poles [18].

\[
B_{sd} = \frac{\mu_0}{l_g} \left( \frac{\sqrt{3}/2N_s}{p} \right) \left( \frac{L_s}{M} i_{sd} \right) + \lambda_{f_d}
\]  

(3.25)

\[
B_{sq} = \frac{\mu_0}{l_g} \left( \frac{\sqrt{3}/2N_s}{p} \right) \left( \frac{L_s}{M} i_{sq} \right)
\]  

(3.26)

From [18], the following expressions are obtained for the torque developed along each axis, where $T_{d\_rotor}$ is the torque on the $d$-axis of the rotor, and $T_{q\_rotor}$ is the torque on the $q$-axis of the rotor.
\[ T_{d,\text{rotor}} = -\frac{p}{2} \left( \pi \frac{\sqrt{3/2} N_s}{p} r l B_{sq} \right) i_{sd} \] (3.27)

\[ T_{d,\text{rotor}} = -\frac{p}{2} \left( \frac{3}{2} \pi \frac{\mu_o}{l_g} r l \left( \frac{N_s}{p} \right)^2 \left( \frac{L_s}{M} \right) i_{sd} \right) i_{sd} \] (3.28)

\[ T_{d,\text{rotor}} = -\frac{p}{2} L_m \left( \frac{L_s}{M} i_{sd} \right) i_{sd} \] (3.29)

\[ T_{d,\text{rotor}} = -\frac{p}{2} (L_s i_{sq}) i_{sd} \] (3.30)

\[ T_{d,\text{rotor}} = -\frac{p}{2} \lambda_{sq} i_{sd} \] (3.31)

\[ T_{q,\text{rotor}} = \frac{p}{2} \left[ \left( \frac{3}{2} \pi \frac{\mu_o}{l_g} r l \left( \frac{N_s}{p} \right)^2 \left( \frac{L_s}{M} \right) i_{sd} \right) + \lambda_{fd} \right] i_{sq} \] (3.32)

\[ T_{q,\text{rotor}} = \frac{p}{2} L_m \left( \frac{L_s}{M} i_{sd} \right) + \lambda_{fd} \] (3.33)

\[ T_{q,\text{rotor}} = \frac{p}{2} [L_s i_{sd} + \lambda_{fd}] i_{sq} \] (3.34)

\[ T_{q,\text{rotor}} = \frac{p}{2} \lambda_{sd} i_{sq} \] (3.35)

The net torque is the sum of the torques on each axis of the rotor, both d and q:

\[ T_{em} = \frac{p}{2} [\lambda_{sd} i_{sq} - \lambda_{sq} i_{sd}] \] (3.36)

By substituting the flux linkage equations into the electromagnetic torque equation, it can be shown that the torque produced by the motor is solely dependent on the flux linkage established.
by the magnets and the $q$-axis current. This also implies that the torque of a motor, is solely controlled with the $q$-axis current.

$$T_{em} = \frac{p}{2} \left[ (L_s i_{sd} + \lambda f d)i_{sq} - (L_s i_{sq})i_{sd} \right]$$  \hspace{1cm} (3.37)$$

$$T_{em} = \frac{p}{2} \lambda f d i_{sq}$$  \hspace{1cm} (3.38)

3.5 Mechanical Equations

As stated previously, the goal of a motor is to transform electrical energy into mechanical energy. To complete the mechanical understanding of the motor the electrical frequency of the driving currents must be related to the mechanical speed of the shaft. Also, the electromagnetic torque should be related to the mechanical torque at the shaft. The following equation relates electrical frequency to shaft speed of a permanent magnet synchronous machine based on the number of pole pairs in the machine [16].

$$\omega_d = \frac{p}{2} \omega_{mech}$$  \hspace{1cm} (3.39)

In practical terms, this means an eight pole machine would make four electrical revolutions to complete one full mechanical revolution. Lastly, it is stated that the acceleration of the shaft is dependent upon the sum of torques and the equivalent inertia of the system.

$$\frac{d}{dt} \omega_{mech} = \frac{T_{em} - T_{load}}{J_{eq}}$$  \hspace{1cm} (3.40)

Here, $T_{load}$ represents the torque of the load applied to the shaft of the motor and $J_{eq}$ represents the total combined inertia of the motor shaft and rotor along with the inertia of the load.
3.6 Motor Constants

A motor’s performance is often described by motor constants given in the datasheet by the manufacturers. While motor constants do not provide an exhaustive description of machine performance, they are typically what is referred to in industry to quantify motor performance. The model developed later in this paper will take into account the motor constants for calculation of the flux linkage established by the magnets, so it is worth developing an understanding of motor constants. There are three values that are typically used as motor constants: EMF constant, torque constant, and flux linkage. All three motor constants are equivalent in base units. This will be demonstrated in the equations below. First, the EMF constant of a motor describes the back EMF of a motor. The units of the EMF constant are typically given in Volts per rpm. This is inherently vague due to the voltage measurement. Some manufacturers measure peak voltage; others measure RMS voltage. All measurements are performed line-to-line due to the inaccessibility of the motor’s neutral point. Second, the torque constant of a motor describes the torque output of a motor per unit current. Torque constants are typically given in Newton-meters per Ampere. Once again, the lack of description concerning the electrical measurement is evident. Lastly, and perhaps the least commonly used among those in industry, is the flux linkage. The flux linkage constant describes the flux linkage established by the magnets in the stator windings. This is more of a motor designer’s constant rather than an industry-employed constant. However, the units of all three motor constants can be equated as described in the following equations. All motor constants can be equated to each other using appropriate proportional coefficients.

\[
[k_e] = \left( \frac{V}{rad/sec} \right) = \left( \frac{kg \ m^2}{A \ sec^2} \right) = (V \ sec)
\]  

(3.41)
This completes the required understanding of the permanent magnet synchronous machine in order to develop a successful simulation of a motor model. The equations developed in this chapter will be employed to construct a model of the permanent magnet synchronous machine in Chapter 6. Next, the magnetic equivalent circuit of a unipolar permanent magnet synchronous machine is presented.
CHAPTER 4: MAGNETIC EQUIVALENT CIRCUIT

This chapter introduces the flux path in a radial flux, non-salient pole permanent magnet synchronous machine. This flux path is described mathematically by employing a magnetic equivalent circuit in which the flux driven across the airgap due to the rotor magnets is examined.

4.1 Flux Path

The path that the magnetic flux takes through a unipolar permanent magnet synchronous machine is described as follows. The flux path is illustrated below in Figure 4.1

![Figure 4.1: Image Illustrating the Flux Path of a Radial Machine [19]](image)

Let us consider the magnet on the face of the rotor as the flux source. Magnetic flux is pulled through the rotor steel and through the magnet. The magnetic flux crosses the air gap and flows through the stator teeth and steel. If the rotor and stator steels are considered to have an infinite
permeability, there is a point of zero magnetic potential at a certain point in each the rotor and stator. Making this assumption allows for evaluation a single pole of the motor. Take $\Phi_m$ as the magnet flux, $\Phi_r$ as the magnet remanence, $\Phi_g$ as the flux in the airgap, and $\Phi_L$ as the leakage flux. The permeances of the system are defined as $P_L$ is the magnet leakage permeance and $P_{m0}$ as the magnet permeance. The reluctances of the system are defined as follows: $R_g$ as the airgap reluctance, $R_{st}$ as the stator tooth reluctance, $R_{ry}$ as the rotor steel reluctance, and $R_{sy}$ as the stator steel reluctance. With these parameters defined, a simple magnetic equivalent circuit of a permanent magnet synchronous machine is developed for analysis.

4.2 Magnetic Equivalent Circuit

With knowledge of the flux paths through the machine, a magnetic equivalent circuit can be developed as follows.
The rotor steel is modeled by $R_{ry}$. The magnetic flux takes two separate paths through the rotor steel as noted in the above flux path figure. Thus, the reluctances appear in parallel. The magnet is modeled as a DC current source of amplitude $\Phi_r$, where $B_r$ is the remanant flux of the magnet and $A_m$ is the area of the magnet face.

$$\Phi_r = B_r A_m$$  \hspace{1cm} (4.1)

The magnet has an internal permeance modeled by $P_{m0}$. Also, because the magnet is not in a shorted magnetic circuit, but rather an open circuit situation, the magnet contains a leakage permeance. The magnetic flux is driven across the airgap, and the reluctance of the airgap is modeled by $R_g$. Next, the magnetic flux flows through the stator tooth; the reluctance of the stator tooth is modeled by $R_{st}$. The flux then splits and takes two parallel paths through the stator.
steel, modeled by $R_{sy}$. The nodes bridging the stator and rotor steels can be said to be at zero magnetic potential, thus the connection is valid.

Figure 4.3: Simplified Magnetic Equivalent Circuit of Radial PMSM

The stator and rotor reluctances are in parallel and thus can be combined into a single reluctance for each steel, making them $R_{sy}/2$ and $R_{ry}/2$, respectively. The airgap reluctance and stator tooth reluctance appear in series and are simply summed together in order to simplify the circuit further. The simplified magnetic equivalent circuit is illustrated in Figure 4.3.

\[
R_{sy} \parallel R_{sy} = \frac{R_{sy}}{2} \quad (4.2)
\]

\[
R_{ry} \parallel R_{ry} = \frac{R_{ry}}{2} \quad (4.3)
\]

\[
R_g + R_{st} = R'_g \quad (4.4)
\]
The internal permeances of the permanent magnet appear in parallel, and can be summed together to form a single permeance in parallel with the flux source.

\[ P_{m0} + P_L = P_m \]  

(4.5)

The remaining reluctances appear in series, and can be summed together to form a single reluctance in parallel with the flux source and internal permanent magnet permeance.

\[ \frac{R_{sy}}{2} + \frac{R_{ry}}{2} + R'_g = R''_g \]  

(4.6)

Applying Kirchhoff’s current law, the flux driven through the airgap can be calculated using series and parallel combinations of the reluctance and permeance along with the flux source.

\[ \Phi_g = \frac{1}{\frac{1}{R''_g} + P_m} \Phi_r \]  

(4.7)

By substituting the equation for remanent flux into the above equation, the flux driven through the airgap can be described in terms of magnetic parameters.
\[
\Phi_g = \frac{1}{\frac{1}{R_g} + P_m} B_r A_m
\]  

(4.8)

This completes the analysis of the magnetic equivalent circuit of a permanent magnet synchronous machine. The preceding derivation shows that the flux driven through the airgap is directly proportional to the remanant flux of the magnet. Together with the understanding of B-H curves from Chapter 2, this allows for an understanding of the connection between material parameters and the magnetic flux driven across the airgap of the machine.
CHAPTER 5: INDUCTANCE CALCULATIONS

This chapter introduces a method for calculating inductances in permanent magnet synchronous machines. As will be discussed later in the chapter, inductance is defined as flux linkage per ampere. It follows that the inductance of permanent magnet synchronous machines is a function of the magnetic material parameters. This chapter attempts to clarify the calculations involved with determining machine inductances in order to apply our knowledge of magnetic material properties to these calculations.

5.1 Basic Inductance Equations

Inductance is can simply be defined as flux linkage per ampere:

\[ \lambda = Li \quad (5.1) \]

Faraday’s Law relates the time derivative of flux linkage to voltage, as shown below.

\[ v = \frac{d\lambda}{dt} \quad (5.2) \]

Substituting the definition of flux linkage into Faraday’s Law and performing the product rule yields the expanded version of Faraday’s Law.

\[ v = L \frac{di}{dt} + i \frac{dL}{dt} \quad (5.3) \]

Flux linkage can be defined as the product of the magnetic flux and the number of turns encircling that flux:

\[ \lambda = \phi N \quad (5.4) \]
Magneto motive force is defined as magnetic flux through a material divided by a material’s permeability. Magneto motive force in magnetic circuits is analogous equated to voltage in electrical circuits.

\[ F = \frac{\Phi}{P} \]  

(5.5)

The above equations allow for a simple introduction to inductance and allow for equating inductance to various parameters as follows.

\[ L = \frac{\Phi N}{i} = \frac{NFP}{i} = N^2 P \]  

(5.6)

Inductance created by flux linkage within a single coil is referred to as self-inductance. Inductance created by flux linkage between two separate coils is referred to as mutual-inductance. A permanent magnet synchronous machine has both self-inductances and mutual-inductances.

5.2 Inductance Components in PMSM

The phase self-inductance of a permanent magnet synchronous machine is considered to be the sum of three terms. \( L_{gap} \) is the airgap component of self-inductance, \( L_{slot} \) is the slot leakage component of self-inductance, and \( L_{end} \) is the end-turn leakage component of self-inductance. The phase self-inductance is defined by the following equation:

\[ L_{ph} = L_{gap} + L_{slot} + L_{end} \]  

(5.7)

Similar to the phase self-inductance, the mutual inductance between phases is also considered to be the sum of three terms. \( M_{gap} \) is the airgap component of mutual inductance, \( M_{slot} \) is the slot
leakage component of mutual inductance, and $M_{\text{end}}$ is the end-turn leakage component of mutual inductance. The mutual inductance between phases is defined by the following equation.

$$M_{\text{ph}} = M_{\text{gap}} + M_{\text{slot}} + M_{\text{end}}$$  \hspace{1cm} (5.8)

Applying the above concepts and equations to the permanent magnet synchronous machine, the flux linkage equations are derived.

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} L_{aa} & M_{ab} & M_{ac} \\ M_{ba} & L_{bb} & M_{bc} \\ M_{ca} & M_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$  \hspace{1cm} (5.9)

Also note that the mutual inductance between phases $a$ and $b$ is the same mutual inductance that appears between phases $b$ and $a$. This is true for all combinations of phases and is described by the following equations.

$$M_{ab} = M_{ba}$$  \hspace{1cm} (5.10)

$$M_{bc} = M_{cb}$$  \hspace{1cm} (5.11)

$$M_{ca} = M_{ac}$$  \hspace{1cm} (5.12)

In order to obtain a full understanding of the relationship between magnetic material parameters and machine inductances, each of the three winding inductance components is described in the next section.

5.3 Airgap Self Inductance Component

To begin describing the airgap self-inductance component, the magneto motive force drop across the airgap, $F_{\text{gap}}$, is defined in Equation 5.13. In this equation, $N_c$ is the number of conductor turns per phase and $i$ is the current in the phase winding.
\[ F_{\text{gap}} = \frac{N_c i}{2} \]  

Defining \( N_p \) as the number of conductor turns per pole reveals that it is half the number of turns per phase.

\[ N_p = \frac{N_c}{2} \quad (5.14) \]

\[ F_{\text{gap}} = N_p i \quad (5.15) \]

Next, the magnetic field in the airgap, \( H_{\text{gap}} \), is defined. Employing the equations introduced above, the magnetic field in the airgap is calculated as follows.

\[ H_{\text{gap}} = \frac{F_{\text{gap}}}{g'} \quad (5.16) \]

The term \( g' \) describes the equivalent airgap. The equivalent airgap is defined in terms of the physical dimensions of the gap as well as Carter’s coefficient, \( K_c \), and the relative recoil permeability of the magnetic material, \( \mu_{\text{rec}} \).

\[ g' = K_c g + \frac{l_m}{\mu_{\text{rec}}} \quad (5.17) \]

Carter’s coefficient describes the reluctance between two concentric cylinders when slotting is introduced. Veinott employs Baillie’s empirical approximation of Carter’s coefficient in the description of the airgap of permanent magnet synchronous machines [20]:

\[ K_c = \frac{5 + s}{5 + s - \frac{s^2}{r}} \quad (5.18) \]

\[ s = \frac{(\text{slot opening})}{g + l_m} \quad (5.19) \]
\[ r = \frac{\text{(slot pitch)}}{g + l_m} \]  \hspace{1cm} (5.20)

\( l_m \) describes the thickness of the magnet, \( g \) describes the length of the airgap. Now that the magnetic field in the airgap is mathematically described, the flux density in the airgap, \( B_{\text{gap}} \), can also be expressed. The flux density in the airgap is simply the magnetic field in the airgap multiplied by the permeability of air:

\[ B_{\text{gap}} = \mu_0 H_{\text{gap}} \]  \hspace{1cm} (5.21)

With this understanding of the flux density in the airgap, the flux per pole can be described as follows, where \( L_{\text{stack}} \) is the length of motor stack and \( p \) is the number of pole pairs:

\[ \Phi_p = B_{\text{gap}} \frac{\pi D}{2p} L_{\text{stack}} \]  \hspace{1cm} (5.22)

It is important to consider the construction of the motor windings if a motor’s winding were wound “multiple-in-hand.” \( T_{ph} \) describes an equivalent number of windings where \( a \) is the number of parallel conductors:

\[ T_{ph} = \frac{N_p 2p}{a} \]  \hspace{1cm} (5.23)

With this knowledge, the airgap self-inductance of a motor winding can be computed:

\[ L_{\text{gap}} = \frac{\Phi_p T_{ph}}{i} \]  \hspace{1cm} (5.24)

In order to determine whether any terms in the above expression are related to the parameters of the magnetic material, the above expression is broken down as follows.

\[ L_{\text{gap}} = \frac{B_{\text{gap}} \frac{\pi D}{2p} L_{\text{stack}} \frac{N_p 2p}{a}}{i} = \frac{\mu_0 H_{\text{gap}} \pi D L_{\text{stack}} \frac{N_p}{a}}{i} \]  \hspace{1cm} (5.25)
Expansion of Equation 5.24 reveals a single parameter which is a property of the magnetic material: $\mu_{rec}$, the relative recoil permeability of the magnetic material. Therefore, the airgap self-inductance of a permanent magnet synchronous machine depends on the relative recoil permeability of the permanent magnet.

### 5.4 Airgap Mutual Inductance Component

The airgap mutual component depends on the geometric layout of the machine’s windings. Once again, the inductance can be defined as flux-turns per ampere. The shape of the flux distribution in the airgap over a full electrical cycle is assumed to be ideal. Figure 5.1 illustrates this. Phase $a$ is aligned with the entire positive flux distribution in the airgap. Phase $b$ is offset by 120° electrical, as it is in permanent magnet synchronous machines. Phase $b$ now encircles part of the positive flux and part of the negative flux. The same is true for Phase $c$. Therefore, the motor designer must have knowledge of the geometric flux distribution in the gap as well as the geometric winding distribution to calculate the flux encircled by phases $b$ and $c$. 
Flux linkage in the mutual winding is calculated using the number of turns of the mutual conductor, \( N_c \), the flux in the airgap, \( B_{gap} \), and the distribution. The distribution term includes the mean airgap diameter, \( D \), the number of pole pairs, \( p \), \( \theta_+ \) as the winding distribution encircling positive flux, and \( \theta_- \) as the winding distribution encircling negative flux.

\[
\lambda = N_c B_{gap} \frac{D}{2p} L_{stack} \left[ (\theta_+) - (\theta_-) \right]
\]

(5.28)

Expanding the flux density term to identify the airgap permeability results in the following equation:

\[
\lambda = N_c \mu_0 H_{gap} \frac{D}{2p} L_{stack} \left[ (\theta_+) - (\theta_-) \right]
\]

(5.29)
Substituting in equation 5.16 into the airgap magnetic field intensity equation results in the following equation:

\[
\lambda = N_c \mu_o \frac{N_p i}{K_c g + \frac{L_m}{2p} L_{stack} \mu_{rec}} \left[ (\theta_+) - (\theta_-) \right]
\]

(5.30)

When comparing equation 5.30 to equation 5.25, it can be seen that the mutual airgap inductance component is proportional to the airgap self-inductance. With this knowledge, a proportionality constant, \( K_{wd} \), is defined which describes the percentage of positive flux encircled by the mutual winding.

\[
M_{gap} = K_{wd} L_{gap}
\]

(5.31)

\[
K_{wd} = \frac{[(\theta_+) - (\theta_-)]}{2\pi}
\]

(5.32)

In this particular case, the mutual winding can be said to encircle 2/3 of the negative flux and 1/3 of the positive flux. This generates a \( K_{wd} \) of -1/3, and expresses that the airgap mutual inductance component of is 1/3 that of the airgap self-inductance component.

\[
M_{gap} = - \frac{L_{gap}}{3}
\]

(5.33)

Therefore, the airgap component of the mutual-inductance is also dependent upon the magnetic material’s relative recoil permeability.
5.5 Slot Leakage Self-Inductance Component

Not all of the conductors of a phase encircle the flux produced by the magnets; some flux leaks through the slots. The slot leakage self-inductance component takes into account the flux leakage across the stator slots. The governing equation of the slot leakage self-inductance component is given below, where $L_{\text{stack}}$ is the length of the stator, $a$ is the number of parallel paths, $N$ is the number of turns per coil, $P_{\text{slot,self}}[k]$ is an array containing the permeance coefficients of the stator slot, and $C[k]$ is the coilside incidence array.

$$L_{\text{slot}} = \frac{\mu_0 N^2 L_{\text{stack}}}{a^2} \sum_{k=1}^{N_{\text{slots}}} P_{\text{slot,self}}[k] C[k]$$  \hspace{1cm} (5.34)

Note that there are no terms concerning the magnetic material in this calculation. Thus, the magnetic material does not affect the slot leakage self-inductance component.

5.6 Slot Leakage Mutual Inductance Component

The calculation of the slot leakage mutual component is quite similar to the calculation of the slot leakage self-component. The equation given below defines the slot leakage mutual component, where $P_{\text{slot,mutual}}[k]$ is an array containing the permeance coefficients of the stator slot, and $C_A[k]$ and $C_B[k]$ are the coilside incidence arrays for each of the phases.

$$M_{\text{slot}} = \frac{\mu_0 N^2 L_{\text{stack}}}{a^2} \sum_{k=1}^{N_{\text{slots}}} P_{\text{slot,mutual}}[k] C_A[k] C_B[k]$$  \hspace{1cm} (5.35)

Once again, note that there are no terms concerning the magnetic material in this calculation. Thus, the magnetic material does not affect the slot leakage mutual component of inductance.
5.7 End Turn Leakage Inductance Components

The end turn leakage components are difficult to calculate accurately. This is due to the complex shape of the end turn itself, the density of conductors, and the relation of the shape to the other end turns within the machine. Luckily, the end turn component is quite small relative to the airgap and slot leakage components of inductance [17]. Hendershot gives an approximate formula for the end turn leakage self-component as follows, where \( R_e \) is the radius of the ideal end-turn, and \( R \) is the mean distance of the coilside:

\[
L_{\text{end}} = \mu_0 N^2 R_e \left[ \ln \frac{8R_e}{R} - 2 \right]
\] (5.36)

Once again, note that there are no terms concerning the magnetic material in this calculation. Hendershot also attempts to derive equations for the mutual term of end turn leakage, however numerous assumptions must be made as to the actual assembly of the machine. Due to the large number of assumptions made in the calculation of this inductance and the relatively small magnitude of this inductance, this paper will ignore the effects of end turn leakage on the total inductance of the machine.

5.8 Synchronous Inductance

The synchronous inductance of a permanent magnet synchronous machine is simply the inductance of a winding in the synchronous reference frame. As discussed in Chapter 3, the dq-axis reference frame rotates synchronously with the rotating magnetic field caused by the stator’s windings. In a non-salient-pole machine the \( d \)-axis and \( q \)-axis inductances are equivalent. Therefore, the synchronous inductance of a machine is simply \( L_d \). This value of inductance is used in the equations derived in Chapter 3 in order describe the performance of a permanent magnet synchronous machine. To begin the derivation of \( L_d \), a three-phase short circuit of the
stator terminals is examined. The phasor diagram of a three-phase short circuit is shown in

*Figure 5.2.*

As presented in [17], Kirchhoff’s voltage law allows for a description of the voltage around the loop of phases E1 and E2 as follows:

\[ E_1 - E_2 + jwL(I_A - I_C) - jwL(I_B - I_A) + jwM(I_B - I_A) - jwM(I_A - I_C) + jwM(I_C - I_B) - jwM(I_C - I_B) = 0 \]  

(5.37)

In this equation, \( L \) is the phase self-inductance, and \( M \) is the phase mutual inductance. The last two terms cancel. Note the current phasor diagram defines phase currents as follows:

\[ (I_A - I_C) = I_1 \]  

(5.38)

\[ (I_B - I_A) = I_2 \]  

(5.39)

\[ (I_C - I_B) = I_3 \]  

(5.40)

Grouping alike terms simplifies the above equation as follows:
\[ E_{12} + jwL(I_1 - I_2) + jwM(I_2 - I_1) = 0 \]  
\[ (5.41) \]

\[ E_{12} + jw(L - M)(I_1) + jw(M - L)(I_2) = 0 \]

At this point, it is important to note the following relationships between the line-to-line voltage and the line-to-neutral voltage of a three-phase motor:

\[ E_{12} = \sqrt{3}E_1 e^{j\pi/6} \]  
\[ (5.42) \]

In addition, due to a balanced three-phase system whose currents all sum to zero, the following relation can be said to be true as well:

\[ I_2 = I_1 e^{j2\pi/3} \]  
\[ (5.43) \]

Substituting the above relationships into equation 5.38 yields the following equation whose voltage and current terms all reference to the same phase:

\[ \sqrt{3}E_1 e^{j\pi/6} + jw(L - M)(I_1) + jw(M - L)I_1 e^{j2\pi/3} = 0 \]  
\[ (5.44) \]

\[ \sqrt{3}E_1 e^{j\pi/6} + jw(L - M)(I_1)(1 - e^{j2\pi/3}) = 0 \]  
\[ (5.45) \]

Applying Ohm’s Law to the above equation results in a mathematical description of phase impedance:

\[ \frac{E_1}{I_1} = \frac{jw(L - M)\left(1 - e^{j2\pi/3}\right)}{\sqrt{3}e^{j\pi/6}} \]  
\[ (5.46) \]

Through simplification, the governing equation of phase impedance can be found:

\[ \frac{E_1}{I_1} = jw(L - M) = jwL_d \]  
\[ (5.47) \]
By canceling the $jw$ terms in 5.47, it can be shown that the synchronous inductance of a phase is simply the self-inductance of the phase minus the mutual-inductance:

$$(L - M) = L_d$$  \hspace{1cm} (5.48)

As described previously in this chapter, the phase self-inductance and mutual inductance is described with the following equations:

$$L = L_{gap} + L_{slot} + L_{end}$$  \hspace{1cm} (5.49)

$$M = M_{gap} + M_{slot} + M_{end}$$  \hspace{1cm} (5.50)

At this point it is convenient to define two new terms, the self-inductance leakage term $L_{\sigma}$ and the mutual-inductance leakage term $M_{\sigma}$:

$$L_{\sigma} = L_{slot} + L_{end}$$  \hspace{1cm} (5.51)

$$M_{\sigma} = M_{slot} + M_{end}$$  \hspace{1cm} (5.52)

In a sinusoidally wound, non-salient-pole permanent magnet synchronous machine, the windings are spaced 120° apart. This leads to the consideration that the mutual-inductance airgap component is the self-inductance airgap component multiplied by cosine 120°:

$$M_{gap} = L_{gap} \cos 120^\circ = \frac{L_{gap}}{2}$$  \hspace{1cm} (5.53)

Substituting the equation 5.53 into equation 5.48 results in the following expression for the synchronous inductance of the permanent magnet synchronous machine:

$$L_d = \frac{3}{2} L_{gap} + (L_{\sigma} - M_{\sigma})$$  \hspace{1cm} (5.54)
With this definition of synchronous inductance, all necessary parameters of the permanent magnet synchronous machine are now known. Using this knowledge, Chapter 6 constructs a model of the permanent magnet synchronous machine.

In conclusion, the derivations in this chapter demonstrate that the only term that the magnetic material affects is the airgap component of self and mutual inductance. However, the relative recoil permeability of permanent magnets is known to be close to unity. For example, the relative recoil permeability of NdFeB is 1.05 [21], and the relative recoil permeability of SmCo is 1.1 [22]. Data provided by the research group at The University of Alabama demonstrates that MnBiCo has a relative recoil permeability of 1, as shown in Figure 2.3. The resulting differences in the airgap component of self-inductance and mutual inductance are minuscule, and can therefore be ignored. The leakage components of the phase and synchronous inductance also remain unchanged due to the lack of dependence on any magnetic material parameters. Thus, the phase and synchronous inductances of a permanent magnet synchronous machine remain unchanged even with the introduction a new magnetic material, assuming the material has a near unity relative recoil permeability.
CHAPTER 6: MODELING THE HARDWARE MOTOR

In this chapter, the motor equations developed in Chapter 3 will be implemented in a computer model of a permanent magnet synchronous machine. The basis for the model of the hardware motor is the Matlab/SIMULINK permanent magnet synchronous machine block.

6.1 Simulink Motor Block/ Equations Modeled

The motor block included with Matlab/SIMULINK is shown below in Figure 6.1. This motor block can be set up to represent a specific hardware motor by establishment of various configuration parameters as will be described later in this chapter. The primary reason this block is employed is the fact that it includes the code necessary to interface SimPowerSystems electrical signals with traditional SIMULINK signals. This is illustrated in the figure by the mnemonic representation of each terminal on the block. The arrow type terminals are typical SIMULINK signals, and the square terminals are SimPowerSystems physical signals. Combining these two types of signals allows full integration of the electrical representation of the motor drive circuitry with the mechanical representation of the motor and its associated load.

![Simulink Model of Permanent Magnet Synchronous Machine](image)

Figure 6.1: Simulink Model of Permanent Magnet Synchronous Machine
Figure 6.2 illustrates the electrical function blocks under the mask of the outer block. As illustrated, Park’s transformation is applied to the terminal voltage of the motor in order to obtain the \(dq\)-winding voltages. The \(dq\)-winding voltages are sent to a block labeled “id, iq”, the output of which is the \(dq\)-winding currents. The currents are then applied to the electromagnetic torque equation and sent out of the block.

\[\text{Electrical Model}\]

Figure 6.3 displays the internal components and connections of the “id, iq” function block. The terminal voltages and shaft speed are passed into blocks labeled “id” and “iq”. These blocks contain the governing equations of \(d\)-axis current and \(q\)-axis current, respectively.

\[\text{dq-Axis Currents}\]
The equations for the $dq$-winding currents are obtained by solving the stator $dq$-winding voltage equations for the respective time derivatives of current. Below, the $q$-axis voltage equation is solved for the time derivative of the $q$-axis current. The resulting equation is modeled in the Simulink block “iq” as shown in Figure 6.4.

\[ v_{sq} = R_s i_{sq} + L_q \frac{d}{dt} i_{sq} - \omega_d (L_d i_{sd} + \lambda_{fd}) \]  
\[ (6.1) \]

\[ \frac{d}{dt} i_{sq} = \frac{v_{sq} - R_s i_{sq} - \omega_d L_d i_{sd} - \omega_d \lambda_{fd}}{L_q} \]  
\[ (6.2) \]

Figure 6.4 q-Axis Current Modeled in SIMULINK

Below, the $d$-axis voltage equation is solved for the time derivative of the $d$-axis current. The resulting equation is modeled in the Simulink block “id” as shown in Figure 6.5.

\[ v_{sd} = R_s i_{sd} + L_d \frac{d}{dt} i_{sd} - \omega_d L_q i_{sq} \]  
\[ (6.3) \]
Finally, the mechanical equations developed in Chapter 4 are modeled in the Simulink permanent magnet synchronous machine motor block. The mechanical equations used in the simulation take into account torques due to both coulomb and viscous friction, both of which were neglected in the initial model of the permanent magnet synchronous machine presented in Chapter 3. The model resulting from inclusion of these additional parameters is shown below in Figure 6.6. Both the shaft speed and position are passed to the output terminals of the block along with select mechanical measurement feedbacks.

\[
\frac{d}{dt} \omega_{mech} = \frac{T_{em} - T_{load}}{J_{eq}} \tag{6.5}
\]

\[
\frac{d}{dt} \omega_{mech} = \frac{T_{em} - T_{load} - T_{friction}}{J_{eq}} \tag{6.6}
\]
To aid in controller feedback and measurement purposes, the motor block is designed with many measurements accessible from outside the block. Measurements such as rotor position and stator abc-winding currents are useful when developing control systems for permanent magnet synchronous machines. Having these quantities available eliminates the need for additional measurement blocks within the simulation. This results in quicker development time and also improves simulation performance. Additional quantities such as rotor speed and developed electromagnetic torque, which aid in motor performance analysis, are also available. All available measurement signals are combined on a Simulink bus and connected to an output terminal. The entire ensemble of available measurement signals is shown in Figure 6.7.
6.2 Motor Parameter Measurements

To configure the computer simulation to represent a specific hardware motor, a series of parameter measurements must first be performed in order populate the appropriate values in the developed model. These parameters include: winding resistance, armature inductance, a motor constant, and the equivalent inertia and friction. IEEE has published a document titled *IEEE Guide for Test Procedures for Synchronous Machines* [23]. This publication establishes a standard for performance and parameter test procedures for synchronous machines, which will be used throughout the ensuing discussion in the remainder of this chapter.

The motor selected as the test bed for these experiments is the DiscoDyne SE1128 [24]. The SE1128 is a permanent magnet synchronous machine, sometimes referred to as an AC
servomotor, or a brushless DC motor. The datasheet rates the SE1128 at 4.8 kW with an EMF-constant of 0.47 V·s/rad. The windings are stated to have a resistance of 0.31 Ω and an inductance of 1.85 mH. The motor standing alone has a moment of inertia of 3.6x10⁻³ kg·m² and a mechanical time constant of 2.9 ms.

6.2.1 Winding Resistance

Winding resistance is a simple measurement. The preferred method of measuring winding resistance is the four-terminal resistance measurement. The four-terminal resistance measurement is more accurate than the traditional two-wire technique because the resistance of the test leads is automatically excluded from the measurement. Because resistance is a function of temperature, it is also important to record the temperature of the windings when performing this measurement. It should be noted that permanent magnet synchronous machines are typically connected in an internal “wye” fashion. This causes the neutral point of the motor windings to be internal to the motor, and thus inaccessible. It is also important to record the resistance measurement between each pair of motor winding terminals to ensure the windings are balanced. *Figure 6.8* illustrates the test circuit utilized for performing the four-terminal resistance test. Table 6.1 contains the results of the four-terminal resistance test for each pairing of the motor’s windings. The resistance test was performed 22.2°C, and the average winding resistance was found to be 0.2632Ω. This measurement confirms the datasheet specification with reasonable error.
6.2.2 Armature Inductance

Measuring the armature inductance of the existing motor is completed by examining the terminal voltage and current under a step change condition with the rotor locked in place. Locking the rotor ensures proper axis alignment and no induced back-EMF. The circuit shown in Figure 6.9 illustrates the test circuit, referred to as a Jones-Bridge [25].
In order to measure the $d$-axis inductance, the rotor’s $d$-axis is aligned and locked with the stator’s $a$-phase. In order to measure the $q$-axis inductance, the rotor’s $q$-axis is aligned and locked with the stator’s $a$-phase. The motor windings are connected to the test circuit as displayed in Figure 6.9. The bridge is then balanced for a DC voltage. The voltage should be high enough to establish a measureable current in the motor windings, but not as large as to burn the motor windings. This means that the potential at the terminals of the integrating voltmeter (in this case acting as a flux linkage-meter) is zero. The switch is then reversed. The voltage waveform resulting from the step change is recorded and integrated yielding a flux linkage. The inductance of the aligned axis is then solved for by the following equations [26]:

\[
\Psi = \int_0^\infty v dt
\]  

(6.7)

\[
L = \left(\frac{R_3 + R_{\text{winding}}}{R_{\text{winding}}}\right) \frac{\Psi}{I_{dc}}
\]  

(6.8)
The results of the inductance measurement and calculation indicate a winding inductance of 1.56 mH, confirming the datasheet specified inductance of 1.85 mH with reasonable error.

6.2.3 Motor Constant Measurements

The EMF-constant of a motor was described in Chapter 4. The measurement of the EMF-constant is a simple test, and is open to some interpretation. Hendershot claims the two most common methods used to calculate the EMF-constant are (1) measuring the peak line-to-line voltage and (2) finding the averaged value of a precisely rectified three-phase back-EMF [17]. This paper will use the peak line-to-line voltage for the purposes of calculating the EMF-constant. The shaft of the motor (device under test) is spun at a known speed. The stator is open-circuited. The peak line-to-line voltage is read from an oscilloscope. Calculations are performed to produce the EMF-constant in units of Volt/rad/s [27]. Figure 6.10 illustrates the back-EMF waveforms recorded while spinning the SE1128 motor at 1000 rpm, 2000 rpm, and 3000 rpm (the motor shaft was spun using a dynamometer and the motor’s windings were open-circuited for this test.)
Figure 6.10: EMF Test at Various Speeds

The resulting EMF-constant calculations are as follows.

\[ k_{e,1000} = \frac{73 \text{ [V]}}{1000 \text{ [rpm]}} = 0.6971 \frac{Vs}{rad} \]  \hspace{1cm} (6.9)

\[ k_{e,2000} = \frac{145 \text{ [V]}}{2000 \text{ [rpm]}} = 0.6923 \frac{Vs}{rad} \]  \hspace{1cm} (6.10)

\[ k_{e,3000} = \frac{219 \text{ [V]}}{3000 \text{ [rpm]}} = 0.6971 \frac{Vs}{rad} \]  \hspace{1cm} (6.11)

The above calculations are averaged together to give the motor’s EMF-constant, as shown in equation 6.12. It should be noted that the measured value is reasonably consistent with the value specified in the manufacturer datasheet.

\[ k_e = 0.6955 \frac{Vs}{rad} \]  \hspace{1cm} (6.12)
6.2.4 Equivalent Mechanical Parameters

The model developed in this chapter requires knowledge of two mechanical parameters: the friction coefficient and the inertia of the system. These parameters are typically provided on the datasheet of a motor. However, because the test setup requires that a mechanical load be attached to the shaft of the motor, the provided values are no longer accurate. The total equivalent damping and inertia of the motor, dynamometer, and instrumentation must be known. In order to simplify the procedures involved, the required parameters have been experimentally measured instead of theoretically calculated.

Viscous friction is one of the rotational losses associated with permanent magnet synchronous machines. These losses occur due to bearing friction and wind resistance and can be described by the following equation:

\[ \tau = B(\omega_2 - \omega_1) \]  

(6.13)

In the above equation, \( \tau \) is the torque on the system, \( \omega_2 \) is the rotational speed of the rotor, \( \omega_1 \) is the rotational speed of the stator, and \( B \) is the friction coefficient. The friction coefficient is given in Newton meter seconds. The stator of the system is fixed, so \( \omega_1 = 0 \).
Figure 6.11: Nominal Shaft Torque Under Steady State Conditions

Figure 6.11 plots the nominal system torques required to maintain a range of motor shaft speeds. Each data point represented on this plot was taken at after the system reached a constant shaft speed. A linear data fit has been performed on the resulting trend yielding the friction coefficient of the entire system:

\[ B = 2.5 \times 10^{-6} \ [Nms] \]  

(6.14)

The final system parameter needed for the computer simulation is the moment of inertia of the system. The moment of inertia of a system is given in kilogram meters squared, and this quantity describes the torque required to accelerate a rotating mass. A common kinematics equation describes the relationship between the moment of inertia and the friction coefficient:

\[ J\dot{\omega} + B\omega = 0 \]  

(6.15)
Another well-known expression relates the ratio of moment of inertia and the friction coefficient to the time constant of a motor:

\[ T_c = \frac{J}{B} \]  \hspace{1cm} (6.16)

To quantify the moment of inertia, a spin-down test is performed on the system. First, the system is spun up to its maximum operating speed; in this case, the speed used was 3000 rpm. The system is then allowed to spin down to a stop under no external forces. The data resulting from this test is shown in Figure 6.12. The time constant of the system is said to be the time the system takes to reach -3dB lower than the initial speed. The data resulting from this test was fitted with an exponential allowing for identification of the mechanical time constant of the system. With the equation 6.16, the equivalent moment of inertia of the system is calculated as follows:

\[ J = 16.67 \times 10^{-6} \text{[gm}^2\text{]} \]  \hspace{1cm} (6.17)
With these system parameters, the computer simulation and model of the permanent magnet synchronous machine is complete. In the next chapter, a brief introduction to motor drives is given; a motor drive is selected for the hardware verification; and this drive is modeled in Simulink.
7.1 Introduction to Motor Drives

A motor drive is a device that allows a user to operate a motor at various speeds and/or torques. Motor drives are a necessary component in modern motion systems that demand precise behavioral control. It is possible to connect a motor to the utility mains without a motor drive; however, this restricts the system to the electrical frequency of the utility grid (e.g., 60Hz in the US). A motor configured in this way is thus restricted to one speed; this speed is given by the following equation:

\[ N = \frac{f \times 60}{p} \]  

(7.1)

In equation 7.1, N is the shaft speed, f is the frequency of voltage applied to the terminals of the motor, and p is the number of pole pairs in the motor. For example, an eight-pole machine has a synchronous speed of 900 rpm. If this eight-pole motor were connected to the utility mains, the speed would be restricted to operating at 900 rpm due to the fixed frequency of the grid. In order to control the speed of a synchronous motor, the frequency of the voltage applied to the motor terminals must be controlled. A motor drive provides this control, and such a system can be thought of as an AC-DC-AC converter. A motor drive takes AC voltage from the grid, converts it to a DC voltage, and then inverts it back into a controllable AC voltage. There are two distinct subsystems within a motor drive: the power stage, and the control stage. The power electronics are the electrical components that make up the high-power circuitry, and the controller is the system that commands specific actions from the power electronics based on the current and desired states of the system.
7.2 Power Electronics

The heart of the motor drive is the power electronic switch. A combination of two switches in the configuration shown in Figure 7.1 is referred to as a switching power pole [28]. One switching power pole operating independently is also called a half-bridge inverter. A system in which two switching power poles operate together and are connected to the same DC bus is called a full-bridge inverter. The circuit diagram for a full-bridge inverter is given in Figure 7.2. A system in which three switching power poles operate together and are connected to the same DC bus is called a three-phase inverter. The circuit diagram for a three-phase inverter is given in Figure 7.3.

![Figure 7.1: Basic Half-Bridge Inverter](image-url)
Three-phase motor drives commonly employ the three-phase inverter topology. This allows for independent control of all three phases connected to the motor. A DC link supplies power to the three-phase inverter. The DC link usually consists of a large, high-voltage capacitor bank. A complete system comprising the three-phase inverter connected to the DC link capacitor is shown in Figure 7.4.
In order to complete the closed control loop, the motor drive must employ several different types of feedback devices. For example, a motor drive generally has current transducers on two of the three motor lines, for current feedback. Current feedback enables the motor drive to close the loop on the current controller. It also requires an external rotary encoder or resolver for shaft position feedback. Shaft position feedback enables the motor drive to operate the commutation loop accurately, as well as close the loop on the velocity and position control loops. A modern motor drive also employs a voltage sensor on the DC link voltage to close the loop on any voltage controllers in the power amplifier unit.

A motor drive must also be able to quickly dissipate energy in order to brake, or slow down, the motor. There are two ways to do this: resistive load bank braking and regenerative braking. Regenerative braking allows the excess energy to flow back into the electrical supply. For utility-connected systems, regenerative braking requires the use of a grid-connected-converter. A grid-connected-converter is a three-phase inverter module with its three-phase lines connected to the grid and its DC lines connected to a DC link. The second way to dissipate power during braking is through the use of a resistive load bank. This type of braking can be realized by a chopping transistor and load resistor in parallel with the DC bus capacitors.

Figure 7.4: Three-Phase Inverter with DC Link Capacitor
7.3 Hardware Motor Drive

As part of this work, a motor drive model has been developed to describe the behavior of the motor drive used in the hardware verification portion of this project. The motor drive fulfilling this role is a Kollmorgen SR85200 paired with the Kollmorgen PA85 power amplifier unit. The PA85 power amplifier unit is a three-phase grid-connected converter. It is designed to convert 208V to 325VDC, and produces a rated output power of 26.4 kW on the DC bus. This particular unit is not outfitted with a regenerative braking unit. Therefore, it is connected to a resistive load bank to handle the braking capabilities of the motor drive. The Kollmorgen SR85200 motor drive is a three-phase AC servomotor motor drive. It accepts a 325VDC input bus from the power amplifier unit. The module has a switching frequency of 8kHz, and is capable of delivering 85 A (continuous) to the motor. This device has a maximum output power of 33.8 kVA and requires a resolver as the motor position feedback device.

The motor drive used in this work employs a voltage PWM control scheme. The core processor runs at 40 kHz. The unit has several nested control loops that allow the user to operate the system in one of several different modes. An illustration of the motor drive control loops is depicted in Figure 7.5. The software application used to program the motor drive control loops is shown in Figure 7.6. The following discussion will describe each of the available control loops used in this system, the individual function of each loop, and the interdependence between the individual control loops.
Figure 7.5: Motor Drive Controller Loops [29]

Figure 7.6: Programming the Motor Drive Control Loops [29]
The innermost loop is the current control loop. The current loop receives its reference value from the output of the commutation loop. The feedback from each of the current transducers is subtracted from each of the respective reference values to determine an error signal. The error signal is sent through a PI controller. The output of this PI controller is referred to as the torque command signal. The torque command signal for each of the three phases is delivered to a PWM generation block. The PWM generation block generates the gate drive signals for each of the six switches of the three-phase inverter module. The current loop updates at 16 kHz.

Working outward, the next loop is the commutation loop. The commutation loop receives its reference signal from the velocity loop, or from a user-input torque reference. The objective of the commutation loop is to generate three appropriately phase shifted signals from a single reference signal. This is accomplished by multiplying the reference signal by $\sin \theta_i$, where $\theta_i$ is the electrical position of the shaft and rotor with index i. The resulting signals are output to the current control loop. The commutation loop updates at 16 kHz.

Again working outward, the next loop is the velocity loop. This loop is not needed if the user is commanding a specific torque. However, this loop is needed if the user is commanding the velocity or position of the motor shaft. The velocity loop takes in either a user input velocity or the command signal from an outer position loop. The feedback signal from the feedback device on the motor shaft is subtracted from the reference signal to produce an error signal. The error signal is then passed through a PI controller, generating a current command signal. The current command signal is then sent to the commutation loop. The velocity loop updates at 4 kHz. The application used to program the Kollmorgen velocity loop is shown below in Figure 7.7.
A model of the motor drive system considered by this work was developed using MATLAB/Simulink. The three-phase inverter module leverages the built-in universal bridge block. The universal bridge block is a configurable switching power pole model. In this instance, the bridge is modeled as a three-phase inverter utilizing IGBT switches. Since the focus of this work is not modeling motor drives, a simple DC voltage supply provides power to the inverter module. The voltage of this source is set to 325 VDC, which is the same voltage the Kollmorgen PA85 power amplifier supplies to the DC link. The DC voltage supply in the
simulation environment can both supply and sink current. This allows the current study to neglect the braking system of the motor drive. The gate drive circuit is also neglected, as the IGBT module is driven by simple Boolean signals within the simulation environment. The three-phase output of the IGBT module is connected directly to the motor terminals. As discussed previously, the motor block provides an array of feedback signals which can be used to operate the controller. In the example shown in Figure 7.8, the rotor angle and three-phase current are the selected feedback signals from the motor.

Figure 7.8: Motor Drive Model

7.5 Motor Drive Controllers

As part of this work, the control loops of the motor drive were also modeled in MATLAB/Simulink. The motor drive controller model implementation is based on the documentation found within the motor drive programming software, as well as the manufacturer documentation for this model of motor drive. A system level overview of the control loops present in this model is shown in Figure 7.9.
The controller begins with a velocity command input which serves as the reference signal for the velocity control loop. The velocity command is passed through an acceleration limiter block, which prevents the controller from seeing step changes. This models the functionality of the motor drive in that the user must define a velocity ramp rate. Next, the signal is passed through a speed limiting block. This models the functionality of the motor drive in that the user must define a maximum rpm. Finally, to ensure unit consistency, the signal is converted from rpm to rad/s. The velocity feedback signal is also input into the control loop. The feedback signal comes from the motor block as a mechanical shaft position. The velocity feedback signal is then subtracted from the velocity command signal, thereby generating a velocity error signal. This error signal is sent through a PI controller modeled after the motor drive controller programming software shown in Figure 7.7. Next, the controller output signal is passed through a current limiting block. This block models the functionality of the motor drive in that the user must program a maximum current. The magnitude of the maximum current is typically given on the nameplate or datasheet of the motor. Limiting the maximum current ensures protection for the motor’s windings and magnets. Finally, the limited command signal is passed through a zero-
order-hold. This hold models the update rate of the velocity loop at 4 kHz. The complete velocity control loop model is shown in Figure 7.10.

![Figure 7.10: Velocity Loop Block](image)

The current command signal leaves the velocity control loop block and enters the commutation loop block. The objective of the commutation loop is to transform a single current command signal into a commutated three-phase current command. In order to accomplish this task, the controller must have knowledge of the current rotor position in electrical radians. The position feedback comes directly from the motor block as a mechanical shaft position. This signal is divided by the number of pole pairs to convert mechanical shaft position to electrical radians, as follows:

$$\omega_e = \frac{p}{2} \omega_{mech}$$  \hspace{1cm} (7.2)

In the above relationship, $p$ is the number of machine poles. Each phase of a three-phase machine is separated by 120°. There is also an initial offset that represents the difference in mechanical to electrical zero position. In the simulation, this offset is represented by the $\pi/2$ term added to the current position feedback and the respective phase shifts. The sum of these angles is passed through a sine block. Each resultant is multiplied by the current command to generate the current command for each of the three phases. The three signals are subsequently combined using a Simulink “multiplexer” object in order to keep the simulation organized. Finally, the three current commands are passed through a zero order hold which changes the
update frequency to 16 kHz. This hold models the update rate of the velocity loop. The complete velocity control model loop is shown in Figure 7.11.

Figure 7.11: Commutation Loop Block

The commutated current command signals leave the commutation loop block and enter the current control loop block. Each of the three current feedback signals also enters the current control loop. The three-phase current feedback signal is subtracted from the three-phase current command in order to generate an error signal for each phase. The error signal is then passed into a PID controller. The output of the PID controller is called the voltage command. The voltage command is sent through a zero-order-hold which changes the update frequency of this block to 16 kHz. This hold models the update rate of the velocity loop. Finally, the voltage command is sent through a voltage limiting block. This voltage limiting block allows the simulation to saturate the voltage applied to the motor terminals. The complete velocity control loop model is shown in Figure 7.12.
Figure 7.12: Current Loop Block

The voltage command signal leaves the current control block and enters the PWM generator block. First, the voltage command is normalized to the voltage of the DC link. In this case, the DC link is being modeled as a DC voltage source, as discussed previously, so there is no need to incorporate a DC link voltage feedback within this block. The voltage command is divided by the DC link voltage through the use of a fixed gain block. Finally, the normalized voltage command is sent to the PWM generator block. The simulated PWM generator block is shown in Figure 7.13. This block generates a triangle wave of amplitude 1 at the switching frequency. Each of the three voltage command signals is then passed through a comparator comparing its value to the reference triangle wave. If the command signal is higher than the reference, the resultant gate signal is high. If the command signal is lower than the reference, the resultant gate signal is low. Each of the three gate signals is also inverted so as to give a total of 6 gate signals. This allows this block to drive the entire set of switches in the three-phase inverter module. The comparison and gate signal generation is illustrated in Figure 7.14. The six gate signals exit the PWM generation block and enter the IGBT bridge discussed previously.
This completes the discussion of the model developed to represent the Kollmorgen SR85200 servomotor drive. In the next chapter, the performance of the entire system, including both the motor and the motor drive, will be examined. In addition, results obtained from hardware experiments will be compared to the simulation output to confirm proper operation of this simulation environment.
CHAPTER 8: SYSTEM LEVEL SIMULATION AND RESULTS

This chapter examines the performance of the model created in Chapter 7, and provides validation of this simulation environment using results of actual hardware experiments. *Figure 7.8* and *Figure 7.9* depict the power stage, the control stage, and the interface signals between these subsystems. The computer simulation of this system is a discrete, fixed time step simulation with a step size of $6.25 \times 10^{-7}$ seconds. This time step allows for reasonable resolution as well as reasonable execution time.

Two types of experiments are compared in this thesis. The first experiment involves spinning the motor at a fixed velocity and placing a step change in load torque on the shaft. This experiment is performed across various shaft speeds and load torques. The second experiment involves spinning the motor at a fixed velocity, applying a load to the shaft, and attempting to accelerate the velocity of the shaft. This experiment is also performed across various shaft speeds and load torques.

In order to obtain reasonable simulation results, the PID controllers of the control system must be tuned. In this work, the Ziegler-Nichols tuning method was employed in simulation for both the current controller and the velocity controller. This tuning method is described in detail in the literature [30] [31]. The results of this controller tuning procedure are evaluated by examining the simulated system’s performance under varying speeds and torques. Several simulations were executed in order to test various combinations of shaft velocity, load torque, and setpoint changes. *Figure 8.1* displays the result of one such simulation. The simulated shaft speed closely follows the reference speed under varying load torque conditions. This demonstrates acceptable controller performance.
With the controller tuned and performing properly, the results of the computer simulations can be compared to the results obtained from hardware experiments. The test setup used for these procedures included the DiscoDyne SE1128 motor connected to the Kollmorgen SR85200 motor drive. A resolver is mechanically connected to the shaft of the SE1128 motor and electrically connected to the Kollmorgen motor drive as a feedback device. The shaft of the SE1128 motor is connected to a machine called a dynamometer. A dynamometer is a machine capable of motoring and loading a revolving shaft, while simultaneously taking precise measurements regarding the mechanical behavior of the system. The dynamometer has its own controller allowing for control over the speed of the shaft or the torque applied to the shaft. The dynamometer also contains the appropriate instrumentation to record shaft speed, torque, and
power. A photograph of the SE1128 motor connected to the dynamometer is shown in *Figure 8.2*.

![Figure 8.2: Hardware Test Configuration with SE1128 Coupled to Dynamometer](image)

The first hardware test performed was conducted as follows. The SE1128 motor was spun to 500 rpm. When the motor reached steady-state, a near step-change in the applied torque was actuated from 0 to 10 Nm. *Figure 8.3* shows the resulting motor performance. The same test procedure was subsequently executed using the computer simulation. *Figure 8.4* shows the simulation output resulting from this scenario, overlaid with the empirical results from the previously-described hardware test. As can be seen from the figure, the computer-simulated system behavior diverges significantly from the behavior of the hardware system. This result led to an exercise in effectively “de-tuning” the PID controllers in the computer simulation. After many trials, the controllers in the simulation were tuned to match the performance of the hardware setup. *Figure 8.5* shows a second overlay of the simulation output and the empirical
results subsequent to this de-tuning procedure. It can be observed from this figure that the agreement between simulation and experiment was significantly improved by this de-tuning procedure.

The next test performed began by commanding the motor to a speed of 1000 rpm. When the motor reached steady-state, a near step change in the applied torque was actuated from 0 to 20 Nm. Figure 8.6 shows the resulting motor behavior, overlaid with the simulation output for this scenario. The final test performed began by commanding a step change in shaft speed from 500 rpm to 1750 rpm. The motor was under a constant 20 Nm load during this procedure. The resulting motor behavior, overlaid with the simulation output for this scenario, is depicted in Figure 8.7. Based on the observation of good agreement between the simulation results and the empirical results in these figures, the simulation environment is considered to be a reasonable representation of the hardware system.
Figure 8.3: Results of Hardware Test
Figure 8.4: Results of Optimized Simulation with Hardware Test Results
Figure 8.5: Results of Retuned Simulation with Hardware Results for Comparison
Figure 8.6: Results of 1000 rpm Test with 0 to 20 Nm Step Change in Load Torque
Figure 8.7: Results of Test at 20 Nm 500 rpm to 1750 rpm
CHAPTER 9: SYSTEM LEVEL SIMULATION OF MOTOR WITH NEW MATERIAL

This chapter introduces the changes necessary to the model of the existing hardware system developed in Chapter 8 in order to incorporate the new magnetic material’s parameters. Incorporating the magnetic material parameters into the model allows for an accurate prediction of machine performance. This model takes into account the remanent flux of the material used to construct the magnets on the rotor of the machine. The remanent flux of the material affects the flux linkage established by the magnets. This is one of the motor constants discussed in Chapter 3 and can be equated to the EMF constant and torque constant of the motor. Chapter 5 introduced the concept that rare earth magnets all have a relative recoil permeability very close to unity; the implication is that the material has little to no impact on the inductance of the machine. Therefore, the new model does not take into account any changes of inductance due to the new material.

A MATLAB script was composed to calculate the new motor’s flux linkage constant in order to properly enter the new parameters into the motor model. This script performs the following function. The user enters the flux linkage constant of the motor with the old magnetic material. The user also enters the remanent flux of the old material. Finally, the user enters the remanent flux of the new material. The script then calculates the new flux linkage constant of the motor with the new magnetic material. The script employs the following equations to accomplish this task. Flux linkage has been previously defined as a flux encircled by a number of conductors. This is described by the following equation where $\lambda$ is the flux linkage, $\phi$ is the magnetic flux, and $N$ is the number of turns of a conductor encircling the flux.

$$\lambda = \phi N \quad (9.1)$$
Assuming minimal losses in the stator teeth, the value of flux in the above equation can be equated to the flux in the airgap derived in Chapter 3.

\[ \phi_g = \frac{1}{\frac{1}{R_g''} + P_m} B_r A_m \]  

(9.2)

If nothing about the motor design changes except for the magnetic material, all terms on the right side of the above equation become independent of magnetic material except \( B_r \), the remanant flux. The old motor constant, \( \lambda_{old} \), is described by the following equation.

\[ \lambda_{old} = \phi_{g,old} N = \frac{1}{\frac{1}{R_g''} + P_m} B_{r,old} A_m N \]  

(9.3)

The new motor constant, \( \lambda_{new} \), is described by the following equation.

\[ \lambda_{new} = \phi_{g,new} N = \frac{1}{\frac{1}{R_g''} + P_m} B_{r,new} A_m N \]  

(9.4)

Next, a constant, \( k_r \), is defined such that \( k_r \) is equal to the constant geometric and magnetic parameters in the above equations.

\[ k_r = \frac{1}{\frac{1}{R_g''} A_m N} \]  

(9.5)

The flux linkage motor constants can then be written as follows:

\[ \lambda_{old} = B_{r,old} k_r \]  

(9.6)

\[ \lambda_{new} = B_{r,new} k_r \]  

(9.7)
The new flux linkage motor constant can then be solved for, canceling out the constant geometric and magnetic parameters:

\[
\lambda_{\text{new}} = \frac{\lambda_{\text{old}} B_{r,\text{old}}}{B_{r,\text{new}}}
\]

(9.8)

This new motor constant is employed in the existing motor model to describe the behavior of the motor with the new magnetic material. In the case of replacing Sm\(_2\)Co\(_{17}\) magnets with MnBiCo magnets the quantitative analysis is as follows:

\[
\lambda_{\text{new}} = \frac{\lambda_{\text{old}}}{0.450 \, [T]} \cdot 0.415 \, [T]
\]

(9.9)

\[
\lambda_{\text{new}} = 0.922 \lambda_{\text{old}}
\]

(9.10)

It can be seen that, with identical motor designs, the flux linkage constant with MnBiCo is 92.2% of the value realized with a Sm\(_2\)Co\(_{17}\)-based design. It follows that the torque constant with MnBiCo will also be 92.2% of the value realized with Sm\(_2\)Co\(_{17}\).

The SE1128 motor studied in this work contains an undocumented permanent magnet. The machine was disassembled in order to access the magnets on the rotor. A sample of the magnetic material was extracted and subsequently analyzed. The material composition was undetermined, however all indications suggest that it is a ferrite permanent magnet. The flux density of a single pole of the rotor was measured to be 1.05 kG, or 0.105 T. This indicates that the existing ferrite magnet is weaker than the proposed MnBiCo magnets. Based on this, it is projected that a motor of the same physical dimensions and properties with MnBiCo magnets will have a torque constant 7.9 times larger than the original motor. The predicted torque
constant of the SE1128 with MnBiCo magnets is 6.403 Nm/A. This value is used in the updated computer simulation described in this chapter.

The simulation environment described in CHX was updated to study the impact of changing the magnetic material from the existing ferrite magnet to MnBiCo. The updated simulation compares the SE1128 motor with the original ferrite magnet to the SE1128 with the proposed MnBiCo magnet. For this simulation exercise, the motor was spun at 500 rpm. When the motor reached steady-state, a near step-change in the applied torque was actuated from 0 to 10 Nm. Figure 9.1 shows the resulting simulated motor performance. These simulation results indicate that the MnBiCo motor tracks the reference speed signal more closely than the original motor design. However, an additional high-frequency oscillatory feature to the shaft speed does appear. As expected, the current delivered to the MnBiCo motor is significantly less than that of the ferrite motor. This is due to the higher torque constant discussed previously. A comparison of current waveforms is shown in Figure 9.2.
Figure 9.1: Comparison of MnBiCo to Ferrite Magnet Motor Performance (Shaft Speed)
The next simulation exercise began by commanding the motor to a speed of 1000 rpm. When
the motor reached steady-state, a near step-change in the applied torque was actuated from 0 to
20 Nm. Figure 9.3 shows the resulting simulated motor performance. Once again, these
simulation results indicate that the MnBiCo motor tracks the reference speed signal more closely
than the original motor design. However, once again, an additional high-frequency oscillatory
feature to the shaft speed does appear. As expected, the current delivered to the MnBiCo motor
is, once again, significantly less than that of the ferrite motor. This is due to the higher torque
constant discussed previously. A comparison of current waveforms is shown in Figure 9.4
Figure 9.3: Comparison of MnBiCo to Ferrite Magnet Motor Performance (Shaft Speed)
Figure 9.4: Comparison of MnBiCo to Ferrite Magnet Motor Performance (Current)
CHAPTER 10: CONCLUSION

In this thesis, the impact of changing the magnetic material in a permanent magnet synchronous machine has been evaluated by theoretical analysis and simulation without requiring the use of finite element analysis.

First, this thesis presented background material regarding rare earth materials. The uses of rare earth materials include an array of applications in consumer, military, and industrial sectors. It was also established that the most powerful and energy-dense permanent magnets are constructed utilizing rare earth materials. The supply chain of rare earth materials begins in foreign nations. This fact has lead the United States to become nearly 100% reliant upon imports to satisfy the increasing demand for rare earth materials. The Department of Energy recognized this trend and began the ARPA-E REACT program in response to this growing challenge. This program focuses on finding alternatives to rare earth materials in critical technologies. In particular, permanent magnet synchronous machines are under investigation due to the widespread use of these machines in electric vehicles and wind turbines. One important challenge for the identification of replacement materials is the existence of a knowledge gap regarding the influence of the magnetic material properties on the performance of permanent magnet synchronous machines based on these materials. This knowledge gap is demonstrated by the lack of publications describing this relationship.

Second, this thesis provided an overview of permanent magnets. This overview included the definition of permanent magnets, a brief discussion on B-H curves, and finally the proposed MnBiCo magnetic material currently being researched as an alternative to rare earth magnets in permanent magnet synchronous machines. Parameters and properties of note were discussed in regards to popular magnetic materials in permanent magnet synchronous machines. A
comparison of Sm$_2$Co$_{17}$, NdFeB, and MnBiCo demagnetization curves resulted in the identification of a favorable remanence flux property for MnBiCo. A comparison of the effects of temperature on Sm$_2$Co$_{17}$, and MnBiCo was also completed showing favorable operation of MnBiCo under high temperatures.

Next, this thesis derived a model of the permanent magnet synchronous machine. The model employed Park’s transformation and the $dq$-axes technique. First, the flux linkages along the $dq$-axes were derived. Next, stator voltage and current equations were developed. Equations governing the production of electromagnetic torque were also provided. The mechanical model of the machine was also derived. Finally, common motor constants such as the EMF-constant, and torque constant were discussed.

Next, this thesis developed an equivalent magnetic circuit of the permanent magnet synchronous machine. This magnetic equivalent circuit was used to describe the flux path through the machine for a single pole. The equivalent circuit was generated based on an ideal flux distribution in the airgap of the machine. Employing the developed equivalent circuit, an expression for flux through the airgap was developed taking into account the magnetic parameters discussed in previous chapters.

Next, this thesis discussed the creation of a computer model of the permanent magnet synchronous machine. The equations developed in previous chapters were compiled into a single MATLAB/Simulink model. A specific, commercially-available motor was selected for performing hardware verification. This motor was then tested for the parameters necessary to populate the model.

Next, this thesis derived expressions for the inductance of the permanent magnet synchronous machine taking into account magnetic material parameters. This investigation was
based on the idea that the magnetic material could affect the inductance of the machine. It was found that the airgap component of inductance, both self and mutual, does depend on the relative recoil permeability of the magnet. Finally, an expression for the synchronous inductance of a machine was derived. This formulation allowed for continued analysis in the $dq$ reference frame.

Next, this thesis described the development of a motor drive control system and circuit. A brief introduction to motor drives was given. A specific, commercially-available motor drive was then selected for hardware verification. The motor drive circuitry was modeled in MATLAB/Simulink using the SimPowerSystems library. The motor drive control system was also modeled in MATLAB/Simulink utilizing a voltage PWM control scheme.

Next, this thesis introduced the system-level computer model of the complete hardware test system. The controllers in the simulation were tuned first for good performance, and then were de-tuned in order to obtain agreement with hardware test results. Both the model and hardware setup were evaluated under various speed and torque commands. The empirical results were subsequently plotted and compared to the simulation output. The end result was an accurate computer simulation which demonstrates good agreement with the behavior of the hardware test setup.

Finally, this thesis demonstrated the performance of the simulated hardware motor when the new MnBiCo magnetic material was introduced into the machine. The process of calculating new motor constants based on the properties of the magnetic material was discussed and implemented in a MATLAB script. The resulting motor constant was employed in the simulation of the system running the same tests as used previously. The resulting motor performance was plotted and discussed. The MnBiCo magnets caused the torque constant of the SE1128 to increase by a factor of 3.95. This resulted in a lower current necessary to maintain a high load.
torque, but also resulted in a back-EMF increase that effectively limited the machine to a maximum shaft speed of 600 rpm with the current motor drive configuration.

10.1 Limitations and Future Work

The models developed in the thesis deliberately ignored any non-linearities and saturation of magnetic materials. The effects of such anomalies have been studied using finite element analysis, and the results have been reported in the literature. As the goal of this thesis was to bypass the need for finite element analysis, these anomalies were ignored. The model in this thesis is also only valid for non-salient pole machines such as machines whose magnets are surface mounted on a smooth rotor. Pole saliency introduces varying inductance in the $dq$-axes; this additional complexity has not been examined in this thesis. Also yet to be considered, and representing an opportunity for future work, is the coercivity of the new magnetic material. As discussed in Chapter 2, if an external magnetic field (such as the magnetic field in the airgap of a machine) of sufficient magnitude is introduced to the magnetic material, the material can become damaged or demagnetized. This and other subtle effects of the magnetic field in the airgap of the new rare earth free machine design have yet to be examined.

10.2 Significance

This thesis completed the necessary derivations of PMSM governing equations in order to determine the effects of the magnetic material on the performance of PMSMs. This thesis showed that the torque constant of a PMSM scales with the remanant flux of the magnetic material on the rotor of the PMSM. Also shown was a negligible, but slight, dependence of machine inductances on the relative recoil permeability of the magnetic material on the rotor of the PMSM.
REFERENCES


