ESSAYS ON YIELD CURVE MODELS WITH MARKOV SWITCHING AND MACROECONOMIC FUNDAMENTALS

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A DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Economics, Finance, and Legal Studies in the Graduate School of The University of Alabama

TUSCALOOSA, ALABAMA

2014
ABSTRACT

This dissertation explores the interaction of the term structure of interest rates and the macroeconomy for the United States and United Kingdom. In particular, using a dynamic factor yield curve model, the three essays of this dissertation investigate the macroeconomic sources of parameter instability in the US and UK term structure. First, this dissertation explores if parameter instability in the term structures is reflected in structural breaks in latent yield curve factors – level, slope, and curvature. I test for a single and for multiple structural breaks. The results indicate that parameter instability in the US term structure is adequately captured by the structural breaks in the level and slope factors. Similarly, there is evidence that structural breaks in the level and curvature factors characterize parameter instability in the UK term structure.

Next, I assume the dynamics of the US term structure follow a two-state Markov process. This allows interest rate dynamics to switch between the two states as frequently as the data dictates. A switching model is proposed which gives macroeconomic insight into an asymmetric monetary policy effect during expansions and recessions. A second proposed switching model provides evidence of a great moderation in the US term structure where there is a dramatic decrease in the volatility of yields.

Lastly, I investigate the interaction of the UK term structure and macroeconomy. In order to establish a definitive one-to-one correspondence between macroeconomic fundamentals and latent yield curve factors, I estimate a dynamic yield curve model augmented with macroeconomic variables. Through impulse response analysis, I find that during the inflation-targeting period for the UK, the curvature factor is directly related to real economic activity. I
then use this established interaction between the term structure and macroeconomy to gain macroeconomic insight into regime changes in the UK term structure. Using Markov-switching dynamic yield curve models, I estimate the term structure and find that periods of low volatility correspond to regimes where real economic activity and monetary policy have a greater effect on the bond market. Periods of high volatility are driven by inflation expectations having a greater influence on bond pricing.
DEDICATION

To my parents Jonah and Clarice Levant, and the educators of P.S. 111, North Birmingham Elementary School, C. J. Going Elementary School, Huffman Middle School, and Alabama School of Fine Arts who made learning fun.
ACKNOWLEDGMENTS

Sir Isaac Newton once said, “If I have seen further it is by standing on ye shoulders of Giants.” Here I would like to acknowledge my “Giants”. I want to thank my family, friends, and colleagues for their continued support throughout my graduate school tenure. I am most indebted to Dr. Jun Ma for taking me under his wing and for the countless hours he dedicated to training me but still giving me the freedom to grow into myself as an Economist. I would also like to express my gratitude to Dr. Junsoo Lee who challenged and ultimately set me upon the path of Econometrics. I thank the rest of my committee: Dr. Robert Brooks, Dr. Daniel J. Henderson, and Dr. Pu Wang for their helpful suggestions and critiques.

Special acknowledgements go to Corey Kline, Shanah Tirado, Tameka Salman, Miesha Williams, the Kline family, Samory Touré Pruitt II, and Daniele and Andrew Dixon who kept my sanity intact with their positive energy, prayers, and distractions. And of course, acknowledgements go to my most vocal supporter, my mother the late Clarice Ann Levant who always believed in how far I could go even when I was not aware of it.

Special thanks go to the Southern Regional Education Board and the late Dr. Billy Helms for the financial support to complete the doctoral degree. I also, thank the staff of the EFLS department for all of the support and help along the way.
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CHAPTER 1

STRUCTURAL BREAKS AND THE EXPLANATORY POWER OF NELSON-SIEGEL FACTORS ON THE TERM STRUCTURES OF THE US AND UK

1.1 Introduction

The term structure of interest rates contains information about a nation’s economic outlook. The calendar-time dimension of the term structure describes the evolution of interest rates as a time series, while the maturity-time dimension describes the evolution of interest rates over bonds’ time to maturity – the yield curve. Information regarding inflation expectations, real business activity and monetary policy can be ascertained from analyzing the time-series properties of the maturity ranges—short, medium, and long—and the yield curve factors—level, slope, and curvature— that heavily influence the magnitude of interest rates for those maturities. Since the yield curve factors have been linked to macro-variables in the literature, establishing a stronger relationship between the yield curve and its factors allows for greater understanding of those variables impact on the yield curve and interest rate dynamics.

The relationship between yield curve factors and macro-variables has been investigated widely in the literature. Stock and Watson (2005) and Dewachter and Lyrio (2006) show the level factor (the long-term yield-to-maturity) closely proxies for inflation expectations. Employing a vector auto-regression (VAR), Bekaert, Cho, and Moreno (2010) show the variation in the level factor is dominated by inflation target shocks, further strengthening this relationship between inflation and the level factor. The slope factor (the long-term yield-to-maturity minus the short-term yield-to-maturity) varies with real business activity, see Harvey
(1988), Chen (1991), and Estrella, Rodrigues, and Schich (2003). Bekaert et. al (2010) shows monetary policy shocks dominate the slope and curvature factor variation. The curvature factor’s link to macro-variables has been more elusive. It has been identified as a coincident indicator of economic activity Modena (2008), a leading indicator Moench (2012) and related to monetary policy Dewachter and Lyrio (2006) and Bekaert et al. (2010). One of the findings of this paper suggests the curvature factor is weakly identified and therefore validates the inability of previous works to establish a consistent relationship between the factor and macro-variables. I utilize the Diebold and Li (2006) factorization of the Nelson and Siegel (1987) yield curve model (henceforth DL model) to explore this connection between macro-variables and the unobserved yield curve factors.

A number of the papers mentioned above employ a VAR framework to account for variation in the factors by “shocking” them with specific macro-variable shocks. This paper takes advantage of the well documented parameter instability of the term structure dynamics in both the United States and the United Kingdom and detects similar parameter instability in specific yield curve factors for both countries using a single and a multiple structural break test. The term structure of interest rates of the United States experienced increased volatility and levels due to unprecedented inflation levels of the 1970s and the monetary policy changes of the late 70s and early 80s with nominal interest rates going as high as 16%. Clarida and Friedman (1984) employ a VAR to investigate the macro-variables responsible for the high nominal rates after the Federal Reserve’s monetary policy switch to targeting monetary aggregates post-October 1979.1 The late 1980s ushered in a period of lower interest rate levels and volatility

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1 Other authors that conclude a break date around this period include Huizinga and Mishkin (1984), Sanders and Unal (1988) and Ma (2008)
known as the Great Moderation that has persisted until the present. Rudebusch and Wu (2007) find evidence of a break in interest rates around 1987.

Likewise, the United Kingdom’s term structure has exhibited parameter instability due to economic phenomena such as the demand and monetary shocks of the 1970s, Margaret Thatcher’s dis-inflationary agenda in the 1980s, and the UK’s exit from the Exchange Rate Mechanism in October 1992. Benati (2008) shows the Great Inflation episode of the 1970s was due in large part to demand non-policy shocks and to a lesser extent supply shocks. Using a Bayesian VAR, he is able to argue that the stability in the UK term structure beginning in the 1990s is due more to the “good luck” hypothesis than “good policy” that seemed to come with Margaret Thatcher’s counter-inflationary agenda in the early 1980s. Bianchi, Mumtaz, and Surico (2009) find evidence that prior to 1992, the UK’s policy rate was largely driven by concerns other than inflation and real activity using a factor-augmented VAR (FAVAR) methodology. But after 1992, with the adoption of inflation targeting, policy shocks explained less of the variability in short term interest rates.

Parameter instability in the term structure has been estimated a number of ways in the extant literature. Sanders and Unal (1988) explore structural breaks in the risk-free interest rate dynamics using the Vasicek (1977) interest rate model by way of the Goldfeld-Quandt regime switching model (GQRSM). Chan, Karolyi, Longstaff, and Sanders (1992) utilize dummy variables to detect a priori breaks in their nesting interest rate model. In accounting for shifts in the term structure, Rudebusch and Wu use a Chow-type test. Ma (2008) employs a number of endogenous structural break tests based on the LM principle to detect structural breaks in the risk-free interest rate dynamics according to the CKLS (1992) specification. Since the focus of this paper is establishing a connection between parameter instability in the term structure and
latent yield curve factors and not rigorously modeling yields or yield factors, the interest rate and
time-series dynamics are characterized as an AR(1) process motivated by the Vasicek (1977) interest rate model which takes the form

\[ dr = a(b - r)dt + \sigma dz \]  

(1)

where \(a\), \(b\), and \(\sigma\) are time invariant and the, risk-free interest rate factor, \(r\), is pulled to a level \(b\) (long-run mean) at rate \(a\). Also, \(dz\) is a Wiener process that is \(N(0,1)\). In this model there is only one source of uncertainty for the interest rate factor – the standard deviation.

This paper contributes to the literature by applying Andrews (1993) SupF test and Bai and Perron (1998, 2003a, 2003b) multiple structural break test to (1) the various maturities that comprise the US and UK term structures and (2) the Nelson-Siegel (NS) yield curve factors for each country. Given the construction of the Nelson-Siegel model, parameter instability in maturity ranges of the term structure should be reflected in parameter instability of those NS yield curve factors that weigh most heavily on those maturities. I find parameter instability in the US term structure is nicely captured by parameter instability in the US level and slope yield curve factors when the SupF test and BP test is employed. This result lends support to the findings of Krippner (2006) and Diebold et al. (2008) that the level and slope factors adequately characterize the US term structure. I find weak evidence of an analogous result for the United Kingdom using a Chow-type F-test where the level and curvature factors sum up the parameter instability of the term structure.

Because of the established connection between the NS factors and macro-variables as discussed above, the structural break in the factors are in accordance with investigated changes in inflation and real activity as a result of monetary policy changes aimed at correcting inefficient levels of those macro-variables. This paper finds that specific NS factors are affected
by the high level and volatility of inflation during the 1970s and 1980s for each country. Also, I
find the dis-inflationary policies of each country affected the term structure differently in each
country. A structural break in the level NS factor is detected for the US which is reflected in
structural breaks detected for longer maturities in the US term structure, while for the UK the
slope NS factor experiences a break which is reflected in shorter maturities experiencing a break
in the UK term structure. Furthermore, both countries’ term structures exhibit a portfolio effect
where simulated confidence intervals are tighter around the break date for longer maturities than
shorter maturities when the $SupF$ test is used, mainly because of the imposition of
homoscedasticity from the $SupF$ test. As a robustness check, I look for evidence of a Great
Moderation in each country by estimating unconditional means and conditional volatilities
before and after the break for each maturity when the Andrews $SupF$ test is employed. There is
evidence of a significant decrease in the unconditional mean and volatility after the estimated
break dates for the term structures of each country.

The paper is organized as follows: Section 2 explains the interest rate models and
structural break tests being employed, Section 3 discusses the data for the US and UK Treasury
yields, and Section 4 discusses the results of the structural break analyses on the term structure
and NS factors for both countries. Section 5 concludes the paper.

1.2 Model and Estimation

In this section I introduce the Diebold and Li (2006) formulation of the Nelson-Siegel
model along with their methodology of extracting the unobserved NS yield curve factors. Then I
discuss Andrews (1993) $SupF$ methodology employed to detect parameter instability in the term
structure and in the NS factors. I use the ordinary least squares properties of the regression
model to derive estimates for the unconditional mean and conditional volatility for the sub-
samples before and after the estimated break date. Finally, I briefly discuss the BP (2003) multiple structural break methodology.

1.2.1 Nelson-Siegel Model

Nelson and Siegel (1987) proposed a model for the yield curve at a point in time based on the Laguerre function which a popular mathematical approximating function. The NS model has the form

\[ y_t(m) = \beta_1 + \beta_2 \left( \frac{1-e^{-\lambda m}}{\lambda m} \right) + \beta_3 e^{-\lambda m} \]  

(2)

where \( \lambda \) is the factor loading parameter that governs the decay rate of the loading factors associated with the slope coefficients \( \beta_2 \) and \( \beta_3 \) while \( m \) represents the maturity. The appeal of the NS yield curve model lies in its ability to estimate different yield curve shapes with only three unobserved, or latent, yield curve factors, \( \beta_1, \beta_2, \) and \( \beta_3 \) referred to as level, slope, and curvature. The NS model’s ability to capture a wide range of yield curve shapes for the countries under investigation—the US and the UK--can be seen in Figure 1.1 and Figure 1.2, respectively.

By construction of the NS model, the three yield curve factors impact the short, medium, and long maturity ranges of the yield curve differently. The level factor has equal weighting on all maturities while the slope factor weighs most heavily on short maturities. Finally, the curvature factor weighs most heavily on medium maturities. Recall, that in addition to the mathematical impact the NS factors have on yield curve shape, I discussed the linkages of the NS factors to macro-economic variables in this paper’s introduction. In summary, the NS yield curve model’s parsimonious nature and the NS factors’ link with macroeconomic variables makes this model a powerful tool in term structure estimation and monetary policy.

In this analysis, I use a modification to the NS model. Diebold and Li’s (2006) dynamic Nelson Siegel (DNS) model
\[ y_t(m) = L_t + S_t \left( \frac{1 - e^{-\lambda m}}{\lambda m} \right) + C_t \left( \frac{1 - e^{-\lambda m}}{\lambda m} - e^{-\lambda m} \right), \quad (3) \]

This parameterization of NS model decreases the coherence between the slope and curvature loading factors. And more importantly for this analysis, it extends the NS model into the time dimension which makes term structure analysis possible.

Diebold-Li point out that since their factorization of the NS model is a highly nonlinear function of the factor loadings, nonlinear least squares (NLLS) can be employed to estimate the parameters \( L_t, S_t, C_t, \) and \( \lambda \). But by fixing the source of the model’s non-linearity at a pre-specified value this allows the regressors, i.e. the factor loadings to be computed for each maturity leaving the coefficients, i.e. factors to be estimated. This can be done efficiently with ordinary least squares (OLS). Diebold and Li hold \( \lambda \) at 0.0609. This value implies that the curvature factor loading is maximized at 30 months. I adopt this methodology with \( \lambda \) fixed at 0.0609 for the US and the UK to attain parameter estimates for \( L_t, S_t, \) and \( C_t \). Once we have the estimates \( \hat{L}_t, \hat{S}_t, \) and \( \hat{C}_t \), we model their dynamics as an AR(1) process.

Because the latent factors are treated as observable in the second step, this two-step approach is subject to the “generated regressor” problem. Diebold and Rudebusch (2013) hypothesize that little information is loss in the two-step approach because there is enough cross-sectional variation for the NS factors to be estimated precisely at each time \( t \). Furthermore, I calculate R-squared values to show how accurately the factors from the two-step approach estimate yield dynamics at each maturity for each time \( t \). The R-squared values are in excess of 0.99 for each maturity for both the US and UK.\(^2\) This supports Diebold and Rudebusch’s conjecture that little is loss in performing the two-step approach.

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\(^2\) The medium 21-month, 24-month, and 30-month yields give the largest R-squared values which corroborates findings in the literature on in-samples estimation using the Nelson-Siegel model.
1.2.2 Andrews (1993) SupF Test

The structural break test I explain in this section is based on Andrews (1993) test and I apply it to both our analysis of the parameter stability of the term structure utilizing the yields, \( y_t \), and the NS factors \( L_t, S_t, \) and \( C_t \). Equation (1) can be expressed in the following discretize form

\[
y_t - y_{t-1} = \alpha + \beta y_{t-1} + \varepsilon_t, \quad (4)
\]

where \( y_t \) are observed yields and the disturbance term, \( \varepsilon_t \), is \( N(0, \sigma^2) \) and \( t = 1, ..., T \). Equation (4) also represents a change of variables from equation (1) where \( \alpha = a \cdot b \) and \( \beta = -a \). In array notation equation (4) is

\[
\Delta y_t = x'_{t-1} \Theta + \varepsilon_t, \quad (5)
\]

where \( x_{t-1} = \begin{pmatrix} 1 \\ y_{t-1} \end{pmatrix} \) and \( \Theta = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \). I assume the same AR(1) dynamics as in Equation (4) for the NS factors when investigating breaks in their dynamics. In this instance, the yield term, \( y_t \), is simply replaced by the row vector \( [L_t, S_t, C_t] \) at each realization \( t \). Assume a break occurs at \( t_b \) then Equation (5) becomes

\[
\Delta y_t = x'_{t-1} \Theta + \varepsilon_t, \quad t = 1, ..., t_b - 1 \quad (6)
\]

\[
\Delta y_t = x'_{t-1}(\Theta + D) + \varepsilon_t, \quad t = t_b, ..., T \quad (7)
\]

where \( D = \begin{pmatrix} d_0 \\ d_1 \end{pmatrix} \) is a dummy vector that takes the value of unity for the time interval \( [t_b, T] \) and zero otherwise. Now let \( x'_{t-1}^{y} = \begin{pmatrix} 1 \\ y_{t-1} \end{pmatrix} \) when estimating the break date in the yields of the term structure and \( x'_{t-1}^{f} = \begin{pmatrix} 1 \\ f_{t-1} \end{pmatrix} \) where \( f_t = [L_t, S_t, C_t] \) when estimating the break date in the NS factors. Then Equations (6) and (7) can be expressed as a single equation for the time interval \( [1, T] \) in matrix notation.
$\Delta y = X' \Theta + X_{tb}^y D + \varepsilon^y,$ \hspace{1cm} (8)

where $\Delta y = (\Delta y_1, ..., \Delta y_T)$, $X^y = (x^y_0, ..., x^y_{T-1})'$, $X_{tb}^y = (0, ..., 0, x^y_{tb}, ..., x^y_{T-1})'$ and 

$\varepsilon^y = (\varepsilon^y_1, ..., \varepsilon^y_T)'$. Similarly, we can express the structural break equations for the NS factors as

$\Delta f = X^f \Theta + X_{tb}^f D + \varepsilon^f,$ \hspace{1cm} (9)

where $\Delta f = (\Delta f_1, ..., \Delta f_T)$, $X^f = (x^f_0, ..., x^f_{T-1})'$, $X_{tb}^f = (0, ..., 0, x^f_{tb}, ..., x^f_{T-1})'$ and

$\varepsilon^f = (\varepsilon^f_1, ..., \varepsilon^f_T)'$. I am interested in detecting structural breaks in the long-run or unconditional mean. The unconditional mean is

$E(\Delta y_t) = -\frac{\alpha}{\beta},$ \hspace{1cm} (10)

for Equation (4) using the fact that $b$ is the long-run mean in Equation (1). In order to detect structural breaks in the unconditional mean, breaks have to be estimated in the constant, $\alpha$, and the slope coefficient, $\beta$. For computational tractability, we assume the break date for both parameters occurs at the same time. Andrews (1993) test is a sufficient structural break test for the purposes of detecting a one-time structural break in the case of an unknown breakpoint. Andrews derives the limiting distribution of the supremum F-statistic over a truncated set of break points used in the testing procedure. A candidate structural break point is detected at period $t_b$ using the test statistic of the form

$SupF = \sup_{t_b \in [\pi T, (1-\pi) T]} F_{t_b},$ \hspace{1cm} (11)

where $\pi$ is the truncation parameter, which we set equal to 0.15 in this investigation. The null hypothesis is of no structural break and can be tested using the asymptotic critical values provided by Andrews. If the null is rejected, the timing of the breakpoint, $t_b$, can be consistently estimated as
\[ \hat{t}_b = \text{argmin}_{t_b \in [\pi T, (1-\pi)T]} (\hat{\epsilon}_{t_b}^\prime \hat{\epsilon}_{t_b}). \] (12)

1.2.3 Estimating Unconditional Mean and Conditional Variance

Using OLS, I am able to estimate parameters before and after the break. Estimates of \( \alpha \) and \( \beta \) before and after the break are used to calculate the unconditional mean for each sub-sample

\[
\hat{\mu}_0 = -\frac{\hat{\alpha}_0}{\hat{\beta}_0}, \quad t = 1, ... , t_b - 1
\] (13)

\[
\hat{\mu}_1 = -\frac{\hat{\alpha}_1 + \hat{\alpha}_0}{\hat{\beta}_0 + \hat{\beta}_1}, \quad t = t_b, ... , T
\] (14)

Since the unconditional mean is a nonlinear function of two random variables, I employ the delta method to calculate standard errors for the mean. The delta method is implemented by linearizing the expression for the mean with a first-order Taylor approximation:

\[
g(\alpha, \beta) = E[-\frac{\hat{\alpha}}{\hat{\beta}} | x_{t-1}] = -\frac{\alpha}{\beta} \approx g(\hat{\alpha}, \hat{\beta}) + g_\alpha(\hat{\alpha}, \hat{\beta})(\alpha - \hat{\alpha}) + g_\beta(\hat{\alpha}, \hat{\beta})(\beta - \hat{\beta}). \] (15)

Taking the variance of both sides of Equation (14) yields

\[ \text{Var} \left( -\frac{\hat{\alpha}}{\hat{\beta}} \right) = g_\alpha^2 \text{Var}(\alpha) + g_\beta^2 \text{Var}(\beta) + 2g_\alpha g_\beta \text{Cov}(\alpha, \beta) \] (16)

where \( \text{Var}(\alpha), \text{Var}(\beta), \text{and} \text{Cov}(\alpha, \beta) \) come from the OLS covariance-variance matrix

\[
\text{Var}[\Theta | x_{t-1}] = \sigma^2 (x_{t-1}^\prime x_{t-1})^{-1}, \quad (17)
\]

\[
E[\frac{\epsilon_t^\prime \epsilon_t}{T-1} | x_{t-1}] = \sigma^2 \quad (18)
\]

With the estimation of the mean being derived from the ratio of two normally distributed variables it is worth noting that Fieller (1932) and Hinkley (1969) point out that the moments of a ratio of normal random variables do not in general exist. Hayya, Armstrong, and Gressis (1975) conclude that the ratio of two normally distributed variables is approximately normally distributed if (1) after applying the Geary-Hinkley Transformation, the coefficient of variation for the denominator is less than 0.39 and the coefficient of variation for the numerator is greater...
than 0.005, (2) without a transformation, the coefficient of variation of the denominator is less than or equal to 0.09 and the coefficient of variation of the numerator is greater than 0.19. Since the sample size in this investigation is relatively large, $T = 372$, the Central Limit Theorem (CLT) can be invoked which ensures our estimates are arbitrarily close to the true parameter values.

1.2.4 Bai and Perron (1998, 2003a, 2003b) Multiple Structural Break Test

The structural break test developed in Bai and Perron (1998, 2003a, 2003b; hereafter BP) is based on the principle of dynamic programming and is the second break test implemented in this paper. In this paper, I do not explain the methodology of the dynamic programming methodology since it has been discussed fully in Bai and Perron (2003a). This section gives the products of the algorithm to determine the estimated breakpoints and parameter estimates for each regime. Consider once again Equation (5) but with $m$ breaks.

$$\Delta y_t = x'_{t-1} \Theta^j + \varepsilon_t, \quad t = T_{j=1} + 1, \ldots, T_j, \quad (19)$$

for $j = 1, \ldots, m + 1$, where $\Theta^j$ is the vector of regression coefficients in the $j^{th}$ regime. This equation is estimated using least squares where the least-squares estimates for $\Theta^j$ are generated by minimizing the sum of squared residuals,

$$S_T(T_1, \ldots, T_m) = \sum_{t=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} (\Delta y_t - x'_{t-1} \Theta^j)^2. \quad (20)$$

Estimated breakpoints are given by

$$\left(\hat{T}_1, \ldots, \hat{T}_m\right) = \arg\min_{T_1, \ldots, T_m} S_T(T_1, \ldots, T_m). \quad (21)$$

The breakpoint estimators correspond to the global minimum of sum squared residuals objective function. In Bai and Perron (1998), they test the null of no structural breaks against the alternative of $m = b$ breaks. Suppose $(T_1, \ldots, T_b)$ is a partition such that $T_i = [T \lambda_i]$ ($i = 1, \ldots, b$). Then the maximum F-statistic considered can be expressed as
\[ \text{Sup}F_T(b) = F_T(\hat{\lambda}_1, ..., \hat{\lambda}_b) \] (22)

where \( \hat{\lambda}_1, ..., \hat{\lambda}_b \) minimize the global sum of squared residuals, \( S_T(T\lambda_1, ..., T\lambda_b) \), under the restriction that \( (\hat{\lambda}_1, ..., \hat{\lambda}_b) \in \Lambda_\pi \), where \( \Lambda_\pi = \{ (\lambda_1, ..., \lambda_b); |\lambda_{i+1} - \lambda_i| \geq \pi, \lambda_1 \geq \pi, \lambda_b \leq 1 - \pi \} \) where \( \pi \) is the truncation parameter.

In this paper I am most interested in the application of Bai and Perron (1998) double maximum statistic and Bai and Perron (1998) \( \text{Sup}F_t(l + 1|l) \) statistic. The double maximum statistics -- \( UDMax \) and \( WDMax \) -- test the null of no structural break against an alternative of an unknown upper bound of breaks I denote as \( M \). The \( UDMax \) statistic is given by

\[ UDMax = \max_{1 \leq m \leq M} \text{Sup}F_T(m), \] (23)

while \( WDMax \) statistic is derived by applying various weights to individual \( UDMax \) statistics to achieve the same marginal p-value across \( m \)-values.

The \( \text{Sup}F_t(l + 1|l) \) statistic tests the null hypothesis of \( l \) breaks against the alternative of \( l + 1 \) break. If the statistic is found to be significant then the additional break caused a significant reduction in the sum of squared residuals. Asymptotic distributions for the double maximum statistic and \( \text{Sup}F_t(l + 1|l) \) statistic are provided in BP (1998, 2003b) as well as critical values for various values of the truncation parameter, \( \pi \), and the maximum number of breaks, \( M \).

1.3 Data

I use end of the month price quotes for US Treasuries as supplied by Francis Diebold. The maturities are the 1, 3, 6, 9, 12, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 month for the period from January 1970 through December 2000. A month is defined as 30.4375 days. Diebold and Li (2006) convert unsmoothed Fama-Bliss forwards rates to unsmoothed Fama-Bliss zero yields. Figure 1.3 gives a three-dimensional mesh plot of the US term structure.
I use end of the month price quotes for UK Treasury yields supplied by Haroon Mumtaz for the same sample period. The UK yield maturities supplied by the BOE were more limited as compared with the Diebold-Li maturities, especially at the short end of the yield curve. The maturities available are the 3, 6, 9, 12, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 month. Figure 1.4 gives a three-dimensional plot of the UK term structure.

In Table 1.1, descriptive statistics for the US Treasury yields are presented. The mean of the yields are increasing in maturity which indicates the typical yields curve in the sample period is upward sloping. Standard deviations show long rates are less volatile than short rates. This supports the idea of a portfolio effect in the term structure posited by Samuelson (1965, 1973,1976), Kessel (1965), Malkiel (1966) and empirically proven by Brooks and Livingston (1990). Finally, displacement autocorrelations show the short rates are less persistent than long rates.

The descriptive statistics for the UK Treasury yields are presented in Table 1.2 and follow the same patterns of the descriptive statistics for the US yields. I also present descriptive statistics of the empirical factors for each country. For the US empirical factors, I calculate the empirical level as the average of the 1-month, 24-month, and 120-month maturities, the empirical slope as the difference between the 120-month and 1-month maturities, and the empirical curvature as double the 24-month maturity minus the sum of the 1-month and 120-month maturities. Because the UK yields data begins with the 12-month maturity and ends with the 108-month maturity, I calculate the empirical level, slope, and curvature as the average of the 12-month, 48-month, and 108-month, the difference between the 108-month and 12-month and double the 48-month minus the sum of the 12-month and 108-month, respectively. The
empirical level factor has the highest persistence, volatility, and mean followed by the empirical slope factor for both the US and UK and then by the empirical curvature factor.

1.4. Empirical Results

The results of the investigation are presented in this section. The methodologies employed detected statistically significant breaks in some maturities and in some of the NS factors for both the US and UK. In the case of Andrews (1993) SupF test confidence intervals of break dates were bootstrapped with there being overlap between the confidence intervals of the maturities and the corresponding NS factors. Using the methodology explained in the previous section, unconditional means were estimated before and after the break date along with standard errors. Finally, the estimated conditional volatility of the model is presented for the pre-break and post-break periods given the various maturities. Figures 1.5 and 1.6 give an idea of how close our NS factors estimated by the methodology outlined in Diebold and Li (2006) are to the empirical factors calculated by the formulas in Section 3 for each country. In both instances, the estimated curvature factor is highly volatile compared with its empirical counterpart.

1.4.1. Single Structural Break Analysis for the US

The Andrews SupF test detects statistically significant breaks at the 5% confidence level for the 48, 84, 96, 108, and 120 month maturities. A break in the 60 and 72-month maturity dynamics is detected at the 10% level. The breaks for the various maturities occur around August, 1981. These results are presented in Table 1.3. When the break test is applied to the AR(1) specification for the change in the NS factor dynamics, a statistically significant break at the 5% level is detected for the curvature factor around March 1982. A marginally significant break at the 10% level is detected for the level factor around October 1981. Since the level factor influences longer-term maturities more than the slope and curvature factors, it is promising
that both the longer-term maturities and the level factor experience a break. The absence of a break in the short and medium-term maturities is in contention with other literature that uses a search-type break test; see Ma (2008) and Sanders and Unal (1988). But the lack of a break for the shorter maturities is not without precedent in the literature. Chan, Karolyi, Longstaff, and Sanders (1992) find they are not able to reject the null of no break in the short rate according to their diffusion model process for short rate dynamics which supports the results of this paper. The finding of no break in the slope factor is corroborated by Estrella, Rodrigues, and Schich (2003) conclusion that the yield-curve spread exhibited stability during the period of the Federal Reserve’s change in monetary policy during the late 1970s and early 1980s. Finally, despite the curvature factor giving the most statistically significant break date, only one medium term maturity, the 48-month, experienced a statistically significant break. But the break date of the curvature factor does coincide with changes in US monetary policy and observed changes in real economic activity in the US.

Since the break dates are estimated with error, I perform 1000 Monte Carlo simulations to derive 95% confidence intervals to place upper and lower bounds on the break date uncertainty. Using the alternative model as the data generating process (DGP), I impose the break date for the maturities. From Table 1.3, the estimated confidence intervals are widest for the shorter maturities’ break dates and tightest for the longer maturity break dates. For example, the six-month maturity has a break date of July 1981 and a confidence interval that spans from October 1975 until March 1993. The 108-month maturity has a break date of August 1981 with a confidence interval that spans January 1981 until April 1982—a much smaller interval than for the six-month maturity. While the narrowing of the confidence intervals is not uniform across maturities the pattern is still evident. This finding is in agreement with Kessel (1965) and
Malkiel (1966) who find that spot interest rates for long maturities have relatively low variances, although variances did not decline monotonically with maturity. Since the estimation of the break date relies on residual variance estimates for each maturity, the portfolio effect of Brooks and Livingston (1990) explains the pattern observed in the narrowing of the confidence intervals over maturities. The portfolio effect states that if forward rates are perfectly correlated, then the variance of the \( m \) period spot rate is an average of forward rate variances. Now, if forward rates are not perfectly correlated, the spot rate variance will be lower than the average of the forward rates. This effect is stronger for longer maturities. This result stems from the fact the \( SupF \) test assumes the residuals are serially uncorrelated and homoscedastic. In the multiple break analysis, the BP procedure relaxes this assumption to allow for serial correlation and heteroskedasticity in the residuals.

There exists some evidence that the timing of the break dates of the maturity ranges—short, medium, and long—coincide with their respective NS factor as shown by maturity break dates falling within the break date confidence interval of the NS factors from Table 1.4. The break date for the curvature factor gives a large confidence interval—November 1977 to April 1988—around its break date of March 1982 despite being significant at the 1% confidence level. This is evidence of the uncertainty in estimating and interpreting the curvature factor. This uncertainty associated with the curvature factor is also evident in the volatility present in the plot of the estimated curvature factor and its empirical counterpart in the third plot of Figure 1.5.

1.4.2 Mean and Volatility Analysis for the US

After imposing the estimated break dates, we estimate equation for each sub-sample before and after the imposed break. Using parameter estimates of each sub-sample, I calculate the unconditional mean with standard errors and the volatility for each sub-sample. Before the
break, short and medium term maturities give unconditional mean estimates that are highly unreliable as evidenced by the generally large standard errors. On a couple of occasions--18-30 month maturities--the unconditional mean was negative, indicating the estimated \( \hat{\beta} \) was greater than zero and therefore not mean-reverting for these particular maturities. The implications of \( \hat{\beta} \) assuming values greater than zero are made clear by rewriting Equation (3):

\[
y_t = \alpha + (\hat{\beta} + 1)y_{t-1} + \varepsilon_t.
\]

With \( \hat{\beta} > 0 \), the slope coefficient for the lagged interest rate causes the dynamics to “explode” and thus renders the interest rate process non-stationary. After the break, the unconditional means for the maturities were stable and statistically significant. These results suggest that the markets immediate response to the policy change of 1979 was with uncertainty reflected in low bond prices and increased volatility.

The volatility for the short and medium range maturities pre-break was generally larger than those of the corresponding maturities’ volatility post-break. The volatilities were about the same for the two sub-samples for the 48-120 month maturities. This further supports the observations that before the break date, markets were experiencing more uncertainty with respect to more liquid assets.

I perform the above analysis and estimate the conditional volatility for the NS factors. Table 1.4 presents the volatility estimates before and after the estimated break date. Recall the level factor proxies for inflation expectations so we would expect a decrease in uncertainty after the break date for the period from October 1981 until the end of the sample period. Instead we see a slight increase. Further investigation shows that this factor is essentially a random walk with an AR(1) coefficient estimate of 0.9987. Any statistical inference for this factor is therefore

---

3 Calculating the unconditional mean of the NS factor via \( -\frac{\alpha}{\hat{\beta}} \) is not informative of the factor dynamics across periods since NS factor means rely upon the time to maturity and the value of the decay parameter.
unreliable. The slope and curvature factors have AR(1) coefficients of 0.9684 and 0.8109, respectively, so inference from the dynamics are more reliable. The volatility of the slope factor is the same before and after the break date. But this break date was not statistically significant. In that context the volatility being the same before and after seems accurate. The volatility of the curvature factor decreased substantially –from 2.01 to 0.88-- after the estimated break date. Since this factor proxies for monetary policy one can interpret this decrease as a diminishing of uncertainty due to monetary policy in the bond markets possibly through more transparent signals from the Fed to financial markets during the period leading into and through the Great Moderation.

1.4.3 Structural Break Analysis for the UK

Employing Andrews *SupF* test again, I detect statistically significant structural breaks for the 12-month maturity with the 18, 24, and 30-month maturities showing marginal significance as presented in . This indicates that for the UK, dis-inflationary policies impacted the short end of the yield curve in contrast to the US where we saw the long end of the yield curve being impacted. The break in the 12-month maturity occurred around July 1992 as shown in Table 1.5. This corresponds closely to the timing of the UK’s departure from the ERM of the EMS. The 18, 24, and 30-month maturity breaks also occurred around this time. A statistically significant break in the slope factor around June 1978 is reported in Table 1.6. While the timing of the break in the slope factor does not contribute meaningful information to the break date for the shorter maturities’ dynamics, the slope factor break is able to give information of the UK’s term structures response to the real business activity in 1970s UK. A break date for the medium and long term maturities was estimated to be around March 1990. Both the level and curvature factors exhibited statistically significant breaks around August 1983 and March 1982,
respectively. Once again, the timing of these breaks gives no information regarding the timing of the breaks in the term structure but the factor break dates do give insight to the dis-inflationary agenda of the 1980s. Soon after becoming Prime Minister in 1979, Margaret Thatcher along with the Chancellor of the Exchequer Geoffrey Howe raised interest rates and lowered taxes to combat inflation. According to King (1995), by 1975 inflation had reached a high of 27%. By 1982 inflation in the UK had decreased to 8.6 percent.

I again simulate confidence intervals for the various break dates using 1000 Monte Carlo simulations for each maturity and the estimated NS factors’ break date. The portfolio effect again produces tighter confidence intervals as time to maturity increases for the yields. The confidence intervals for the UK NS factor break dates are tighter for all factors than the confidence intervals for the US NS factors. The curvature factor’s break date has an associated 95% confidence level of May 1975 through June 1994. This large confidence interval suggests the break date for the curvature factor is unreliably estimated as was the case for the US curvature factor break date. The third plot of Figure 1.6 shows the high volatility of the estimated curvature factor which contributes to the large confidence interval simulated.

1.4.4 Mean and Volatility Analysis for the UK

Here I discuss the unconditional mean and conditional volatility estimates of the UK term structure and the conditional volatility estimates of the UK NS factors. Unlike the unconditional mean estimates of the US, the unconditional mean estimates for the UK were reliably estimated for each sub-sample in Table 1.5. Pre-break estimates for the unconditional mean are increasing with time-to-maturity which is expected. The medium maturities are not afflicted with negative values or large standard errors as was the case with the US. Standard errors were relatively small for all unconditional means. The unconditional means decreased by 44.5% on average before and
after the estimated break date for the various maturities. This is a clear indication that the disinflationary policies put forth were having the desired effect on the entire UK term structure. Recall the short term maturities of the UK term structure experienced a statistically significant break and that break date occurred around July 1992 which is close to the UK’s departure from the ERM on September 16, 1992. And medium and long term maturities experienced a marginally significant break around March 1990 which is close to the UK’s entry into the ERM in October 1990. The goal of the ERM was to reduce exchange rate volatility and increase monetary stability through restricting exchange rates to a lower and upper bound. From the results, joining the ERM lowered the long end of the yield curve. Exiting the ERM increased the slope of the yield curve, aiding in growth in economic activity to help the UK out of recession.

Volatility estimates across the break date followed the same pattern that was observed with the US. After the break volatility estimates had substantially decreased for all maturities. The differences in the magnitudes for the maturities across the break date suggests that the disinflationary policies implemented in the UK had a greater effect of decreasing volatility in the term structure than in the United States. Recall for the US the volatility of the longer maturities was indistinguishable across the break date.

The conditional volatility of each factor followed decreased after the estimated break date with the curvature factor experiencing the greatest decrease in volatility. Applying the macro-factor interpretation discussed earlier in the paper to the UK NS factors, uncertainty due to inflation expectation decreased after 1985.

1.4.5 Multiple Structural Break Analysis for US

I employ the Bai and Perron (2003) procedure to estimate multiple structural breaks in the term structure and in the NS yield factors. I apply the procedure to detect potential breaks in
the constant and AR(1) coefficient \( z_t = \{1 f_{t-1}\} \) as in the single break analysis. I allow up to 5 breaks \( (M = 5) \) and set the truncation parameter \( \varepsilon = 0.15 \). I also allow for heteroskedasticity in the error structure. The results are presented in Table 1.7.

Using the \( SupF_t(l + 1|l) \) statistic and the BP sequential procedure, there is no evidence of a structural break for the 1-36 month maturities. For each of these maturities the \( SupF_t(1|0) \) statistic is insignificant, indicating no structural break according to the BP sequential procedure. Since the sequential procedure can have low power in the presence of multiple structural breaks, I employ the BP double maximum procedure as a robustness check. The \( UDMax \) statistic is insignificant for each of the 1-36 month maturities. The \( WDMax \) statistic is significant for the 1-12 month maturities and insignificant for the 15-36 month maturities. Recall that the \( SupF_t(1|0) \) and \( UDMax \) statistics were insignificant for the 1-12 month maturities, so despite the significance of the \( WDMax \) statistic it appears these maturities are subject to a single structural break. The sequential BP procedure results indicate a single break for the 48-120 month maturities as confirmed by the significant \( SupF_t(1|0) \) statistic for the maturities.\(^4\) The double maximum statistics further support a single break for the longer-term maturities with significant \( UDMax \) and \( WDMax \) statistics. The break occurs around October 1981 for the 48-120 month maturities. This is break date is very near the August 1981 break date estimated with the Andrews \( SupF \) test.

I next apply the BP sequential procedure to the NS yield curve factors of the US term structure to investigate if the parameter instability present in the term structure is reflected in the yield curve factors in terms of the number of breaks and the break dates. The results are presented in Table 1.8. The only \( SupF_t(l + 1|l) \) statistic that is significant for the level factor is significant at the 5% level.

\(^4\) The 48-72 month maturities show significance at the 10% level while the 84-120 month maturities have a break that is significant at the 5% level.
the $SupF_t(1|0)$. This result suggests a single break for the level factor. The significance of both the $UDMax$ and $WDMax$ statistics support the result from the BP sequential procedure. The break in the level factor is estimated to occur around November 1981 –close to the timing of the break of the long term maturities. The slope factor yields insignificant results for the $SupF_t(1|0)$ test indicating no structural break according to the BP sequential procedure. The double maximum test yields mixed results with the $UDMax$ statistic being insignificant but the $WDMax$ statistic being significant. I still conclude no structural break since the $SupF_t(2|1)$ statistic is also insignificant. No break in the slope factor corroborates my finding of no structural break in the short-term maturities of the US term structure. The curvature factor yields highly significant results for both the BP sequential procedure and the BP double maximum test with an estimated break date of April 1982. Unfortunately, it does not appear to capture the parameter instability of term structure especially the parameter instability of the medium-term maturities which it weighs most heavily on. The break date corresponds to the end of the monetary policy experiment of the Federal Reserve but once again it does not correspond to any of the maturity break date estimates.

In summary, the level and slope yield curve factors capture the parameter instability of the US term structure pretty accurately in terms of number of structural breaks and the timing of the breaks. The level factor exhibited one break around the same time as the break in the long term maturities. The slope factor exhibited no break as did the short term and medium term maturities. These findings suggest by targeting monetary aggregates between October 1979 and October 1982, the Fed caused a disruption in the outlook of inflation, i.e. inflation expectations but did not cause such a disruption in real economic activity as concluded by Estrella et. al (2003). The results of the structural break analysis for the US term structure in this paper greatly
corroborate the findings of Krippner (2006) and Diebold et. al (2008) that the US term structure is adequately characterized by just the level and slope NS factors.

1.4.6 Multiple Structural Break Analysis for UK

Keeping all the testing specifications the same as in the US case, I apply the BP sequential procedure to detect multiple structural breaks in the UK term structure. The results of the procedure are presented in Table 1.9. The $SupF_t(1|0)$ statistic is significant for all maturities except the 6 and 120-month maturities, while all $SupF_t(2|1)$ statistics are insignificant indicating a single break for each maturity. All $U_{DMAX}$ and $W_{DMAX}$ statistics are significant. The 3-30 month maturities share an estimated break date of September 1992 while the 36-108 have a break date of May 1990. These break date results support the findings of the single break analysis using the Andrews $SupF$ test earlier in the paper.

Unlike in the case of the US term structure and NS yield curve factors where there was clear evidence of parameter instability in the term structure being captured in the level and slope factors by the BP sequential procedure, the evidence is less clear for an analogous result for the UK term structure and NS factors. Results are presented in Table 1.10. The $SupF_t(1|0)$ statistic is marginally significant for the level factor with an estimated break date of February 1985. This break date is much earlier than the estimated break date of the long-term yields. The slope factor gives a similar result. The $SupF_t(1|0)$ statistic is significant and the $SupF_t(2|1)$ statistic is insignificant, indicating a single break for the slope factor. The estimated break date at is July 1977 much earlier than the September 1992 break date of the short term yield. Finally, the curvature factor yields significant statistics for both the $SupF_t(1|0)$ and $SupF_t(2|1)$ tests, indicating two structural breaks for the curvature factor. The break dates are March 1983 and March 1990. The number of structural breaks and estimated break date do not capture the
estimated break dates of the medium-term maturities. The BP sequential procedure gave very little support to the idea that parameter instability in the UK term structure is reflected in the UK NS factors. Since I know the break date of the various maturity ranges, my next analysis uses these dates as candidate break dates for UK NS yield curve factors to see if a relationship between the UK term structure and NS factors does exist.

I perform a Chow-type F-test to investigate if an analogous result holds for the UK where a few or all NS factors accurately describe the parameter instability in the term structure. I impose the estimated break date of the maturity ranges on the factor that weighs most heavily on that range, i.e. short maturities – slope, medium maturities – curvature and long maturities – level. Figure 1.11 presents the results of this analysis. Imposing a break date of May 1990 for the Chow test on the level factor gives a p-value of 0.0023, so the null of no break point can be rejected in favor of the alternative of a break point occurring in May 1990. The Chow test on the slope factor when a break date of September 1992 is imposed gives a p-value of 0.2880, indicating the null cannot be rejected. The Chow test yields a p-value of 0.0013 when a break date of September 1992 is imposed for the curvature factor, strong evidence of a possible break at this date. This analysis gives some evidence that the parameter instability in the UK term structure may be captured by the parameter instability in the level and curvature factors.

1.5 Conclusion

This study utilizes Andrews (1993) single structural break test and Bai and Perron (1998, 2003a, 2003b) multiple structural break test to detect parameter instability in the US and UK term structures and in the Nelson-Siegel factor dynamics for each country. I assume the interest rate and factor processes follow an AR(1) process motivated by the Vasicek interest rate model. For the US, the single structural break test detected a statistically significant break in long-term
yields and a marginally significant break in the US level yield curve factor around the same time as the monetary policy experiment of the Federal Reserve. Short and medium term yields do not experience a significant break. The US slope factor also experiences no break during the sample period. The single break test suggests the short-term yields of the UK term structure experienced a statistically significant break and correspondingly the UK slope yield curve factor experienced a significant break but not temporally close as in the case for the US long term maturities and level factor. The break date of the UK short-term yields corresponds to the UK exiting the Exchange Rate Mechanism while the break date of the slope factor is around the time Margaret Thatcher was elected prime minister. I simulate confidence intervals for the break dates. These confidence intervals show evidence of a portfolio effect in the term structures of both countries. Break dates for long-term maturities are estimated more precisely than short or medium term maturities evidenced by tighter confidence intervals. Multiple structural break test results for the US indicate parameter instability in the US term structure is captured by parameter instability in the level and slope yield curve factors. This finding is in accordance with results of Krippner (2006) and Diebold et al. (2008). The multiple break test for the UK does not give an analogous result where parameter instability in the term structure is represented by parameter instability in some or all yield curve factors. But when I impose the term structure break dates of the maturity ranges on the respective factors and perform a Chow-type F-test, there is some evidence of such a result. According to the results of the Chow test, parameter instability in the UK long-term yields is captured by level factor and parameter instability in short and medium-term yields are captured by the curvature factor.
1.6 References


Table 1.1
US Treasury Yields Descriptive Statistics

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<th>$\hat{\rho}(30)$</th>
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Note: We define the Level as $(y(1)+y(24)+y(120))/3$, the Slope as $y(1)-y(120)$ and Curvature as $2y(24) - (y(1)-y(120))$

Table 1.2
UK Treasury Yields Descriptive Statistics

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Note: We define the Level as $(y(3)+y(24)+y(120))/3$, the Slope as $y(3)-y(120)$ and Curvature as $2y(24) - (y(3)-y(120))$
Table 1.3

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Notes:
*Significance at the 10% level.
**Significance at the 5% level.
Confidence intervals and standard errors are contained in parentheses.
Table 1.4

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Notes:
** Significance at the 5% level.
***Significance at the 1% level.
The empirical level had a break date of 1981:07 and the empirical curvature had a break date of 1982:03 at the 10% and 5% confidence levels, respectively.
Table 1.5  

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<td>(1.30)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>1984:06*</td>
<td>12.28</td>
<td>0.64</td>
<td>6.28</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1983:12-1984:12)</td>
<td>(0.77)</td>
<td>(4.07)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:  
*Significance at the 10% level.  
** Significance at the 5% level.  
***Significance at the 1% level.  
Confidence intervals and standard errors are contained in parentheses.
Table 1.6  

<table>
<thead>
<tr>
<th>Factor</th>
<th>Break Date</th>
<th>$\hat{t}_b$ &lt; $t$</th>
<th>$t \leq \hat{t}_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>1985:01***</td>
<td>1.24</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(1984:08-1985:02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>1977:05***</td>
<td>2.90</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(1977:01-1977:12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Curvature</td>
<td>1983:03***</td>
<td>10.40</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>(1980:09-1985:07)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
** Significance at the 5% level.
***Significance at the 1% level.
The empirical level had a break date of 1990:03, empirical slope had a break date of 1978:08 and empirical curvature had a break date of 1982:03 at the 10%, 1% and 1% confidence levels, respectively.
Table 1.7


| Maturities | $SupF_T(1)$ | $SupF_T(1|0)$ | $SupF_T(2|1)$ | $UD_{max}$ | $WD_{max}$ | Breakdates |
|------------|-------------|---------------|---------------|------------|------------|------------|
| 1          | 3.63        | 8.21          | ------        | 10.71      | 20.23***   | ------     |
| 3          | 3.56        | 6.21          | ------        | 11.07      | 14.38**    | ------     |
| 6          | 4.03        | 6.92          | ------        | 11.47      | 13.67**    | ------     |
| 9          | 3.76        | 6.36          | ------        | 7.43       | 14.51**    | ------     |
| 12         | 2.89        | 5.71          | ------        | 7.32       | 14.34**    | ------     |
| 15         | 3.27        | 5.28          | ------        | 6.66       | 10.63      | ------     |
| 18         | 3.89        | 6.34          | ------        | 6.34       | 9.70       | ------     |
| 21         | 4.21        | 6.94          | ------        | 6.94       | 9.78       | ------     |
| 24         | 3.72        | 6.34          | ------        | 6.71       | 9.39       | ------     |
| 30         | 3.38        | 5.67          | ------        | 6.67       | 9.79       | ------     |
| 36         | 4.70        | 7.88          | ------        | 7.88       | 10.51      | ------     |
| 48         | 6.02**      | 10.71*        | ------        | 10.72*     | 11.61*     | 1981:10    |
| 60         | 5.14*       | 9.31          | ------        | 9.31       | 12.37*     | 1981:10    |
| 72         | 5.60*       | 10.64*        | 8.56          | 10.64*     | 11.90*     | 1981:10    |
| 84         | 6.26**      | 12.23**       | 8.10          | 12.23**    | 12.32*     | 1981:09    |
| 96         | 6.88**      | 13.23**       | 8.15          | 13.28**    | 13.28**    | 1981:10    |
| 108        | 7.17**      | 13.95**       | 8.19          | 13.95**    | 13.95**    | 1981:10    |
| 120        | 7.82***     | 15.98**       | 9.07          | 15.98**    | 15.98**    | 1981:10    |

**Notes:**
*Significance at the 10% level
**Significance at the 5% level.
***Significance at the 1% level

This table gives results for Bai and Perron (2003) multiple structural break test with the following specifications:

$z_t = \{1 y_{t-1}\}; q = 2; p = 0; \varepsilon = 0.15; M = 5.$

The estimated break date for the 1-36 month maturities was around September/October 1981 which is very close to what was found with the single structural break analysis.

Table 1.8


| Factors | $SupF_T(1)$ | $SupF_T(1|0)$ | $SupF_T(2|1)$ | $UD_{max}$ | $WD_{max}$ | Breakdates |
|---------|-------------|---------------|---------------|------------|------------|------------|
| **Level** | 5.79*       | 12.23**       | 11.36         | 13.12**    | 20.93***   | 1981.11    |
| **Slope** | 3.15        | 6.29          | ------        | 6.94       | 13.61**    | 1980.04    |
| **Curvature** | 12.10***    | 19.12***      | 4.52          | 19.12***   | 20.76***   | 1982.04    |

**Notes:**
*Significance at the 10% level
**Significance at the 5% level.
***Significance at the 1% level

This table gives results for Bai and Perron (2003) multiple structural break test with the following specifications:

$z_t = \{1 f_{t-1}\}; q = 2; p = 0; \varepsilon = 0.15; M = 5.$
## Table 1.9
Multiple Break Results: UK Treasury Yields (1970:02 – 2000:12)

| Maturities | SupF_T(k) | SupF_T(1|0) | SupF_T(2|1) | U_Dmax | W_Dmax | Breakdates |
|------------|-----------|-----------|-----------|--------|--------|------------|
| 3          | 4.08      | 28.16***  | 5.42      | 28.16***| 28.16***| 1992.09    |
| 9          | 6.67**    | 38.00***  | 2.92      | 30.80***| 30.80***| 1992.09    |
| 12         | 5.42      | 31.86***  | 5.54      | 26.08***| 26.08***| 1992.09    |
| 15         | 5.00***   | 23.73***  | 6.64      | 23.73***| 23.73***| 1992.09    |
| 18         | 5.38*     | 20.05***  | 8.49      | 20.05***| 20.05***| 1992.09    |
| 21         | 5.13*     | 14.77**   | 6.74      | 14.77** | 14.77** | 1990.05    |
| 24         | 5.38*     | 14.58**   | 7.58      | 14.58** | 14.58** | 1990.05    |
| 30         | 5.20*     | 13.63**   | 6.66      | 13.63** | 13.63** | 1990.05    |
| 36         | 5.52*     | 12.75**   | 6.52      | 12.68** | 15.16** | 1990.05    |
| 48         | 5.20*     | 12.18**   | 6.95      | 12.18** | 16.45** | 1990.05    |
| 60         | 5.20*     | 12.18**   | 6.95      | 12.18** | 16.45** | 1990.05    |
| 72         | 5.20*     | 12.18**   | 6.95      | 12.18** | 16.45** | 1990.05    |
| 84         | 5.20*     | 12.18**   | 6.95      | 12.18** | 16.45** | 1990.05    |
| 96         | 5.20*     | 12.18**   | 6.95      | 12.18** | 16.45** | 1990.05    |
| 108        | 5.20*     | 12.18**   | 6.95      | 12.18** | 16.45** | 1990.05    |
| 120        | 5.20*     | 12.18**   | 6.95      | 12.18** | 16.45** | 1990.05    |

Notes:
*Significance at the 10% level
**Significance at the 5% level.
***Significance at the 1% level

This table gives results for Bai and Perron (2003) multiple structural break test with the following specifications:
\[ z_t = \{ 1 y_{t-1} \}; \quad q = 2; \quad p = 0; \quad \epsilon = 0.15; \quad M = 5. \]

The estimated break date for the 1-36 month maturities was around September/October 1981 which is very close to what was found with the single structural break analysis.

## Table 1.10

| Factors   | SupF_T(k) | SupF_T(1|0) | SupF_T(2|1) | U_Dmax | W_Dmax | Breakdates |
|-----------|-----------|-----------|-----------|--------|--------|------------|
| Level     | 8.86***   | 11.37*    | -------   | 13.26**| 21.15**| 1985.02    |
| Slope     | 25.58***  | 23.11**   | 4.36      | 23.11**| 23.11**| 1977.07    |
| Curvature | 11.44***  | 17.20***  | 23.77***  | 17.20**| 22.95**| 1983.03, 1990.03 |

Notes:
*Significance at the 10% level
**Significance at the 5% level.
***Significance at the 1% level

This table gives results for Bai and Perron (2003) multiple structural break test with the following specifications:
\[ z_t = \{ 1 y_{t-1} \}; \quad q = 2; \quad p = 0; \quad \epsilon = 0.15; \quad M = 5. \]
Table 1.11

<table>
<thead>
<tr>
<th>Factors</th>
<th>F-stat</th>
<th>p-value</th>
<th>Breakdate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>6.18***</td>
<td>0.0023</td>
<td>1990:05</td>
</tr>
<tr>
<td>Slope</td>
<td>1.25</td>
<td>0.2880</td>
<td>1992:09</td>
</tr>
<tr>
<td>Curvature</td>
<td>6.74***</td>
<td>0.0013</td>
<td>1992:09</td>
</tr>
</tbody>
</table>

Notes:
*Significance at the 10% level
**Significance at the 5% level.
***Significance at the 1% level
Figure 1.1: US Yield Curves

Figure 1.2: UK Yield Curves
Figure 1.3: US Term Structure


Maturity (months)  Time

Yields (percent)
Figure 1.4: UK Term Structure
Figure 1.5: US Smoothed Nelson-Siegel Factors
Figure 1.6: UK Smoothed Nelson-Siegel Factors
2.1 Introduction

The yield curve often contains useful information about real economic activity and inflation. For example, the level factor (the long-term yield-to-maturity) is often argued to be closely related with inflation expectations, while the steepness or the slope factor (the long-term yield-to-maturity minus the short-term yield-to-maturity) has been shown to vary with the business cycles and is heavily influenced by monetary policy (see Evans and Marshall (1998), and Wu (2002)). The most recent monetary policies, such as Operation Twist conducted by the Federal Reserve Bank in an attempt to lower the long-term interest rate and raise the short-term rate, directly work on the yields curve and serve as a great example of how the yield curve, instead of just one single policy rate–federal funds rate–is expected to have a significant impact on the economy. As such, it is important to correctly model the yield curve to understand better its interactions with business cycles, and the monetary policy transmission mechanism through its impact on the yield curve.

One popular approach to modeling the yield curve in the literature is to impose no-arbitrage conditions and derive the yields curve based on latent factors. The majority of research in this area works on the affine class of models for the purpose of tractability. Duffie and Kan (1996) and Dai and Singleton (2000) work out a general class of the affine term structure model which encompasses some of the early models such as Vasicek (1977) and Cox, Ingersoll, and
Ross (1985). Despite being appealing theoretically, these models in general forecast poorly, likely due to their restrictive nature, as pointed out by Duffee (2002).

A second class of models rectified the shortcomings of the first class in a novel way. Employing the relationship from expectations theory, Nelson and Siegel (1987, hereafter NS) was able to model forward rates directly with a three latent factor model and derive the yield curve. The three latent factors represent the level, slope, and curvature of the yield curve. Unlike the no-arbitrage affine models, the NS model greatly improved forecasting across bond maturities and has become very popular, in particular among the central banks. Moreover, recent work by Coroneo, Nyholm and Vidova-Koleva (2011) finds that the NS model is close to being arbitrage-free when applied to the US market, although it does not explicitly impose these restrictions.

Diebold and Li (2006, hereafter DL) extended the NS model to the time dimension by allowing the three latent factors of the NS model to be time-varying. This dynamic Nelson-Siegel (DNS) model’s forecast has been shown to outperform vector autoregressive models and dynamic error correction models. Diebold, Rudebusch, and Aruoba (2006, hereafter DRA) integrate observed macroeconomic variables to the DNS model and study the important interactions of the NS factors and the macroeconomic variables. Diebold, Li, and Yue (2008) further model the yield curves in a set of countries and extract the global yield factors that appear to explain a large portion of the global yield curves movements. The re-interpretation of the NS model by DL emphasizes the factor structure of the NS model. Their extension of the original

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5 See, for example, the BIS (2005) report that points out the central banks in a number of countries including France, Germany, and Spain all use the NS model to model the yield curve.

6 Svensson (1994) introduced a second curvature factor, and thus a second loading parameter \( \lambda \), to the NS framework to better estimate longer termed maturities, a noted weakness of the NS model. Christensen, Diebold, and Rudebusch (2009) included a second slope factor as well as a second curvature factor to derive an arbitrage-free class of NS models.
NS model to the dynamic NS model (DNS) shows that the NS model is essentially a particular factor model that captures the whole yield curve movement through a few latent factors with a particular set of loading parameters. In this way the DNS model is closely related with the dynamic factor model (DFM) that has often been applied to macroeconomic and finance data, such as the work by Stock and Watson (1991) that extracts an economic coincident index from a set of economic variables. Recently, Jungbaeker, Koopman, and Van der Wel (2014) use cubic splines to introduce smoothness in factor loadings of a DFM for the US term structure.

The interest rate dynamics of the term structure has been subject to frequent regime changes (see Bansal and Zhou (2002)). Although some regime changes are results of obvious changes in monetary policy as in the Volcker era and obvious changes in business cycle conditions such as the oil supply shock of the 1970s, there are many other regime changes that are due to more frequent business cycle fluctuations and often indirectly observed changes in the financial markets. As a result, it is important to capture these regime changes in order to capture more accurately the dynamic movements of the yield curve.

In the context of the NS model, Koopman, Mallee, and Van der Wel (2010) include the loading parameter $\lambda$ as a time-varying latent factor to be estimated along with the three time-varying latent factors via the extended Kalman filter, as a way to allow the time variation in the parameters. They also introduce time-varying volatility in the form of a GARCH process into the DNS framework. Wong, Lucia, Price and Startz (2011) study the connection of the yield curves in the US and Canada, and identify an exogenous structural break in the NS model that reveals a weaker correlation between the yield curves in these two countries after Canada changed its monetary policy and switched to the explicit inflation targets in 1991. While Startz and Tsang (2010) incorporate Markov regime switching into an unobserved components model
of the yield curve to account for regime changes of the yield curve. As an alternative modeling approach to the exogenous type of breaks, Markov regime switching proposed in Hamilton (1989) has the advantage that the underlying breaks can be reoccurring and stochastic in nature. Markov regime switching also has been successfully introduced to the DFM by Chauvet (1998), and Kim and Nelson (1998) that generalize the work of Stock and Watson (1991) to allow Markov regime switching in extracting an economic coincident index from a set of macroeconomic variables.

We contribute to the literature by introducing and thoroughly evaluating regime-switching factor loadings and regime-switching volatility in the dynamic Nelson-Siegel model. In our models, regimes are characterized by a latent Markov switching component—the fourth latent factor. We apply a Markov switching component to the loading parameters of the factors as well as the factors’ volatility. Comparisons between the models are made by presenting goodness-of-fit statistics and AIC/BIC values. We also implement the Likelihood Ratio (LR) tests to investigate if our models are statistically different from the baseline linear DNS model. Although both models are found to be statistically different from the baseline model, the root mean square error (RMSE) analysis shows the model with the loading parameter switching yields the smallest RMSE across the short, medium, and long maturity ranges and in terms of overall fit. This model also gives the minimum AIC/BIC values of all models under consideration. Recently, Yu and Salyards (2009) and Yu and Zivot (2011) apply the DNS model to modeling corporate bond yields and they find that the optimal \( \lambda \), which determines the loading parameters of the NS factors, changes as one goes from modeling investment to speculative grade bonds. Their results corroborate our findings in general.
In light of recent discussions about potential interactions between the interest rates factors and the macro-economy, we investigate the relationship between the extracted factors from our DNS models and the observed macroeconomic variables. We find that our interest rate factors, which are extracted separately from the macroeconomic variables, are closely related with the macro-economy. Specifically we find the level factor is strongly correlated with the inflation expectation, and the slope factor appears to be counter-cyclical, which is consistent with the finding by Wu (2002) that the slope factor is related with monetary policy. Furthermore, in the regime-switching DNS model we find that the loading of the slope factor on the yield curve is larger during recessions than expansions. This seems to suggest an asymmetric effect of the monetary policy on the yield curve over business cycles.

This paper is organized as follows. Section 2 describes the baseline linear dynamic Nelson-Siegel model and the regime-switching DNS model, and the estimation procedure via Kalman filter (KF) and the Kim algorithm (KA) is briefly discussed. Section 3 describes the data. Section 4 presents and discusses the estimation results. Section 5 concludes.

2.2 Models and Estimation

In this section we introduce the baseline dynamic Nelson-Siegel (DNS) model. The appeal of the DNS model lies in its extension to the time dimension. We also introduce our regime-switching extensions of the DNS models and the estimation technique used.

2.2.1 The Dynamic Nelson-Siegel Model

The Diebold and Li (2006) factorization of the NS model is given by

\[ y_t(m) = y_t(m; F_t, \lambda) = L_t + S_t \frac{1 - e^{-\lambda m}}{\lambda m} + C_t \left( \frac{1 - e^{-\lambda m}}{\lambda m} - e^{-\lambda m} \right) \]  

(1)
where \( F_t = (L_t, S_t, C_t)' \) is a vector representing level, slope, and curvature of the yield curve, for given time \( t \), maturity \( m \), and constant \( \lambda \), the factor loading parameter. This is the baseline DNS model in our analysis.

The shape of the yield curve comes from the factor loadings and their respective weights in \( F_t \). From Equation (1), the factor loading associated with \( L_t \) is assumed to be unity for all maturities and therefore influences short, medium, and long-term interest rates equally. The loading factors for \( S_t \) and \( C_t \) depend on both maturity and the loading parameter. For a given \( t \), the slope factor loading converges to one as \( \lambda \downarrow 0 \) (or \( m \downarrow 0 \)) and converges to zero as \( \lambda \rightarrow \infty \) (or \( m \rightarrow \infty \)). The curvature factor loading converges to zero as \( \lambda \downarrow 0 \) (or \( m \downarrow 0 \)) and as \( \lambda \rightarrow \infty \) (or \( m \rightarrow \infty \)) for a given \( t \).

Since we are interested in the loading parameter’s effect on yields, we use the limit analysis above to understand the asymptotic behavior of the yield curve. The yield curve converges to \( L + S \) as \( \lambda \downarrow 0 \) and converges to \( L \) as \( \lambda \rightarrow \infty \) for a given \( t \). These limiting values indicate that without the loading parameter the yield curve is flat and with extreme values for the loading parameter the yield curve would become flat. So both “reasonable” values for \( \lambda \) and the level factor are responsible for the wide range of non-flat yield curve shapes within an NS framework.

2.2.2 DNS Model Estimation

We adopt the DRA state-space framework to model each variant of the NS model in this paper. Our measurement equation models the time-series process of the yields according to the
latent factors and takes the form

\[
\begin{bmatrix}
y_t(m_1) \\
y_t(m_2) \\
\vdots \\
y_t(m_N)
\end{bmatrix} =
\begin{bmatrix}
1 & \frac{1-e^{-\lambda m_1}}{\lambda m_1} & \frac{1-e^{-\lambda m_1}}{\lambda m_1} & \cdots & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \cdots & \frac{1-e^{-\lambda m_N}}{\lambda m_N} \\
1 & \frac{1-e^{-\lambda m_2}}{\lambda m_2} & \frac{1-e^{-\lambda m_2}}{\lambda m_2} & \cdots & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \cdots & \frac{1-e^{-\lambda m_N}}{\lambda m_N} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
1 & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \cdots & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \cdots & \frac{1-e^{-\lambda m_N}}{\lambda m_N}
\end{bmatrix}
\begin{bmatrix}
L_t \\
S_t \\
C_t
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_t(m_1) \\
\varepsilon_t(m_2) \\
\vdots \\
\varepsilon_t(m_N)
\end{bmatrix}
\] 

or expressed in matrix notation as

\[
y_t = \Lambda(\lambda)F_t + \varepsilon_t, \quad \varepsilon_t \sim MN(0, G), t = 1, \ldots, T,
\] 

with \(y_t\) representing the \(N \times 1\) vector of yields, \(N \times 3\) factor loading matrix \(\Lambda(\lambda)\), \(3 \times 1\) latent factor vector \(F_t\), and \(N \times 1\) yield disturbance vector \(\varepsilon_t\) (or so-called measurement errors of the yields). The diagonal structure of \(G\) implies that measurement errors across maturities of \(y_t\) are uncorrelated and is a fairly standard assumption in the literature. The transition equation, which models the time series process of the latent factors, can be expressed by the vector autoregressive process of order one

\[
\begin{bmatrix}
L_t - \mu_{Lt} \\
S_t - \mu_{St} \\
C_t - \mu_{Ct}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & 0 & 0 \\
0 & a_{22} & 0 \\
0 & 0 & a_{33}
\end{bmatrix}
\begin{bmatrix}
L_{t-1} - \mu_{Lt} \\
S_{t-1} - \mu_{St} \\
C_{t-1} - \mu_{Ct}
\end{bmatrix} +
\begin{bmatrix}
\eta_{Lt} \\
\eta_{St} \\
\eta_{Ct}
\end{bmatrix}
\]

equivalently expressed in matrix notation as

\[
F_t = (I - A)\mu + AF_{t-1} + \eta_t, \quad \eta_t \sim MN(0, Q), t = 1, \ldots, T,
\] 

with \(3 \times 1\) mean vector \(\mu\), \(3 \times 3\) coefficient matrix \(A\), and \(3 \times 1\) factor disturbance matrix \(Q\). In this investigation we assume a diagonal coefficient matrix based on the findings of Diebold et al. (2006) and Christensen, Diebold, and Rudebusch (2011) who show the off-diagonal elements of the \(A\) matrix are not statistically relevant to modeling the term structure. We also assume \(Q\) follows a diagonal structure implying the factor disturbances are uncorrelated. Once again we look to Diebold et al. (2006), which finds the off-diagonal elements of the factor covariance
matrix to be marginally significant. Furthermore, they note that when the model is estimated with the restriction that the factor covariance matrix be diagonal, the point estimates and standard errors of the $A$ matrix are little changed from when the model is estimated with no restriction on $Q$.

Since the DNS state space model is linear in latent factors, we are able to use the KF to estimate the latent factors conditional on past and contemporaneous observations of the yields. The KF procedure is carried out recursively for $t = 1, ..., T$ with initial values for the latent factors and their variances being the unconditional mean and unconditional variance, respectively. If we define $f_{t|t}$ as the minimum mean square linear estimator (MMSLE) of $F_t$ and $v_{t|t}$ as the mean square error (MSE) matrix, then $f_{1|0} = \mu$ and $v_{1|0} = (I - A)^{-1}Q$. With observation $y_t$ and initial values $f_{1|0}$ and $v_{1|0}$ available, the KF updates the values for $f_{t|t}$ and $v_{t|t}$ using the equations

$$f_{t|t} = f_{t|t-1} + K_t e_{t|t-1},$$

$$v_{t|t} = v_{t|t-1} - K_t \Lambda(\lambda) v_{t|t-1},$$

where $e_{t|t-1} = y_t - \Lambda(\lambda) f_{t|t-1}$ is the predicted error vector, $ev_{t|t-1} = \Lambda(\lambda) v_{t|t-1} \Lambda(\lambda)' + G$ is the predicted error variance matrix and $K_t = v_{t|t-1} \Lambda(\lambda)' ev_{t|t-1}^{-1}$ is the Kalman gain matrix.

The next period $t + 1$ MMSLE of the latent factors and associated variance matrix conditional on yields $y_1, ..., y_t$ are governed by the prediction equations

$$f_{t|t-1} = (I - A) \mu + Af_{t-1|t-1},$$

$$v_{t|t-1} = Av_{t-1|t-1}A' + Q.$$
Denote $\theta$ as the system parameter vector. The parameters to be estimated via numerical maximum likelihood estimation are $\theta_{DNS} = \{A_{ij}, G_{ij}, Q_{ij}, \mu, \lambda\}$. We represent the log-likelihood function as

$$\ell(\theta) = -\frac{NT}{2} \ log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \ log |e_{t}| - \frac{1}{2} \sum_{t=1}^{T} \ e_{t}'(e_{t})^{-1}e_{t}. \quad (9)$$

The function $\ell(\theta)$ is evaluated by the Kalman filter through a quasi-Newton optimization method for the purposes of maximization without inverting the Hessian matrix of this 28-parameter system.

The values for $f_{t|t}$ and $v_{t|t}$ from the last iteration of the KF are used as initial values in the recursive algorithm to obtain smoothed values of the unobserved factors. Iterating the following two equations backwards for $t = T - 1, T - 2, \ldots, 1$, gives the smoothed estimates:

$$f_{t|T} = f_{t|t} + v_{t|t} A(\lambda)'v_{t+1|t} (f_{t+1|T} - \Lambda(\lambda)f_{t|t} - \mu), \quad (10)$$

$$v_{t|T} = v_{t|t} + v_{t|t} A(\lambda)'v_{t+1|t} (v_{t+1|T} - f_{t+1|T})v_{t+1|t}^{-1} A(\lambda)v_{t|t}. \quad (11)$$

These smoothed estimates provide a more accurate inference on $f_t$ because it uses more information from the system than the filtered estimates.

2.2.3 DNS Model with Regime-Switching Loading Parameter

In sub-section 2.1 we established that $\lambda$ and $L_t$ determines the shape of the yield curve. Thus changes in interest rate levels are determined by both $\lambda$ and $L_t$, given other factors. Realizing that keeping $\lambda$ fixed across the sample period may be a source of model mis-specification in the literature (see Diebold and Li (2006) Diebold et. al (2006), and Xiang and Zhu (2013) ), Koopman et al. (2010) treat $\lambda$ as a time-varying latent factor of the model to be estimated in the same fashion as the latent NS factors of the model.

We model $\lambda$ as a regime-switching parameter that influences interest rate levels according to the realized state. We assume the term structure follows a two-state regime
switching process for computational tractability of our model. Investigating ex-post real interest
rates, Garcia and Perron (1996) assume interest rates follow a three-state regime switching
process. And using a reversible jump Markov chain Monte Carlo (RJMCMC) procedure, Xiang
and Zhu (2013) estimate two distinct regimes for the term structure.

We propose to treat the loading parameter \( \lambda \) as a regime switching parameter solely
determined by the realized state, \( S_t \) of the yields. The latent Markov component \( S_t \) is governed
by a two-state Markov process and we denote the states simply as 0 or 1 corresponding to the
term structure being in the low or high regime respectively. The loading matrix \( \Lambda(\lambda) \) in
Equation (3) is replaced by \( \Lambda(\lambda_{S_t}) \) and the resulting measurement equation is
\[
\mathbf{y}_t = \Lambda(\lambda_{S_t}) \mathbf{F}_t + \varepsilon_t.
\]
Take note that since we are not including \( \lambda \) in \( \mathbf{F}_t \), the observation vector of yields is still
linear with respect to our latent factor vector. This new measurement equation along with our
transition equation from Equation (4) constitutes the dynamic Nelson-Siegel with Markov-
switching lambda (DNS-MSL) model for estimation with regime-switching the loading
parameter.

2.2.4 The DNS Model with Regime-Switching Factor Volatilities

In most of the empirical literature on term structure modeling, a constant volatility is
assumed in the time-series of interest rates. Like modeling the DNS model with constant loading
parameter, a constant volatility over time may be a source of model misspecification for
estimating the term structure. A few papers investigate time-varying volatility in the context of
the DNS model. Bianchi et al. (2009) employ a VAR augmented with NS factors and macro-

factors featuring time-varying coefficients and stochastic volatility. Koopman et al. (2010)
estimate yield disturbances according to a GARCH specification to introduce a time-varying
variance.
We modify the DNS model by introducing regime-switching volatility to the factor disturbances in the transition equation. Equation (3) represents the measurement equation of this model and after replacing $\eta_t$ with $\eta_{S_t}$ the new transition equation is

$$F_t = (I - A)\mu + AF_{t-1} + \eta_{S_t}, \quad \eta_{S_t} \sim MN(0, Q(S_t)), t = 1, ..., T, S_t = 0,1.$$  

The state-space model comprising the measurement equation from equation (3) and volatility switching transition equation with $\eta_{S_t}$ substituted into equation (4) will be referred to as the DNS-MSV model.

2.2.5 Regime-switching Model Estimation

In this sub-section we will show that the Kim algorithm (KA) allows for efficient estimation of parameters through the Kalman Filter (KF) and accurate inference of the realized states through a methodology developed by Hamilton (1989, 1990). But before we outline filter, the DNS-MSL state-space model can be represented in its entirety as

$$y_t = \Lambda(\lambda_{S_t})F_t + \varepsilon_t, \quad \varepsilon_t \sim MN(0, G), t = 1, ..., T,$$

$$F_t = (I - A)\mu + AF_{t-1} + \eta_t, \quad \eta_t \sim MN(0, Q), t = 1, ..., T, \quad (12)$$

$$\lambda_{S_t} = \lambda_0(1 - S_t) + \lambda_1 S_t, \quad S_t = 0,1,$$

and the DNS-MSV model in its entirety is

$$y_t = \Lambda(\lambda)F_t + \varepsilon_t, \quad \varepsilon_t \sim MN(0, G), t = 1, ..., T,$$

$$F_t = (I - A)\mu + AF_{t-1} + \eta_{S_t}, \quad \eta_{S_t} \sim MN(0, Q(S_t)), t = 1, ..., T, \quad (13)$$

$$\eta_{S_t} = \eta_0(1 - S_t) + \eta_1 S_t, \quad S_t = 0,1,$$

where for both models the transition probabilities between states are governed by the entries of the matrix

$$\begin{bmatrix}
p_{00} = p & p_{01} = 1 - p \\
p_{10} = 1 - q & p_{11} = q
\end{bmatrix}$$
where \( p_{ij} = \Pr[S_t = j | S_{t-1} = i] \) with \( \sum_{j=0}^{1} p_{ij} = 1 \) for all \( i \).

The estimation of the parameters of the model according to the KA is very similar to the KF procedure explained for the non-switching case. Recall the latent factors for the DNS-MSL and DNS-MSV models are the NS factors and the unobserved state, \( S_t \). We initialize the NS factors and their variances as in the non-switching case. To initialize the unobserved state, \( S_t \), we need \( \Pr[S_0 = j | \psi_0] \) where \( j = 0, 1 \) and \( \psi_t \) refers to information up to time \( t \). This expression is the steady state or unconditional probability of being in the low regime which is given by the formulas

\[
\pi_0 = \Pr[S_0 = 0 | \psi_0] = \frac{1-p}{2-p-q},
\]

\[
\pi_1 = \Pr[S_0 = 1 | \psi_0] = \frac{1-q}{2-p-q},
\]

where \( p \) and \( q \) are defined in the above transition probability matrix. Given realizations of the NS factors at \( t \) and \( t-1 \) when \( S_{t-1} = i \) and \( S_t = j \), the KF can be expressed as

\[
f^{(i,j)}_{t|t} = f^{(i,j)}_{t|t-1} + K^{(i,j)}_{t} e^{(i,j)}_{t|t-1},
\]

\[
v^{(i,j)}_{t|t} = v^{(i,j)}_{t|t-1} - K^{(i,j)}_{t} \Lambda(\lambda)v^{(i,j)}_{t|t-1},
\]

\[
e^{(i,j)}_{t|t-1} = y_t - \Lambda(\lambda)f^{(i,j)}_{t|t-1}
\]

\[
ev^{(i,j)}_{t|t-1} = \Lambda(\lambda)ev^{(i,j)}_{t|t-1} + \Sigma \epsilon
\]

\[
f^{(i,j)}_{t|t-1} = (I - A)\mu_j + Af^{(i,j)}_{t-1|t-1}
\]

\[
v^{(i,j)}_{t|t-1} = Av^{(i,j)}_{t-1|t-1}A' + \Sigma \eta.
\]

where \( K^{(i,j)}_{t} = v^{(i,j)}_{t|t-1} \Lambda(\lambda)'(ev^{(i,j)}_{t|t-1})^{-1} \) is the Kalman gain

The efficiency of the KA arises from collapsing the \( (2 \times 2) \) posteriors \( f^{(i,j)}_{t|t} \) and \( v^{(i,j)}_{t|t} \) into two single-state posteriors
\[ f_t^j = \frac{\sum_{i=0}^{1} \Pr[S_{t-1}=i, S_t=j|\psi_t] f_{t|i}^{(i,j)}}{\Pr[S_t=j|\psi_t]}, \quad (22) \]

and

\[ v_t^j = \frac{\sum_{i=0}^{1} \Pr[S_{t-1}=i, S_t=j|\psi_t] \{v_{t|i}^{(i,j)} + (f_{t|i}^{(i,j)} - f_{t|i}^{(j,i)}) (f_{t|i}^{(j,i)} - f_{t|i}^{(i,j)}) \}}{\Pr[S_t=j|\psi_t]}, \quad (23) \]

by taking weighted averages over states at \( t-1 \). Following Hamilton (1989, 1990), the Kim (1994) algorithm is a consequence of Bayes’ theorem which we can use to get the previous single-state posteriors results. Starting with the joint distribution of our states, we have

\[ \Pr[S_t = j, S_{t-1} = i|\psi_t] = \frac{\Pr[y_t, S_t = j, S_{t-1} = i|\psi_{t-1}]}{\Pr[y_t|\psi_{t-1}]} \]

\[ = \frac{f(y_t|S_{t-1}=i, S_t=j, \psi_{t-1}) \times \Pr[S_{t-1}=i|\psi_{t-1}]}{\Pr[y_t|\psi_{t-1}]} \quad (24) \]

The two terms in the numerator and the probability in the denominator can be put in terms of known quantities from our estimation model. The conditional density \( f(y_t|S_{t-1}=i, S_t=j, \psi_{t-1}) \) is obtained based on the prediction error decomposition:

\[ f(y_t|S_{t-1}=i, S_t=j, \psi_{t-1}) = (2\pi)^{-N/2} |\mathbf{e}_{t|i}^{(i,j)}| \exp \left\{ -\frac{1}{2} \mathbf{e}_{t|t-1}^{(i,j)'} \left( \mathbf{e}_{t|i}^{(i,j)} \right)^{-1} \mathbf{e}_{t|i}^{(i,j)} \right\} \]

and

\[ \Pr[S_t = j, S_{t-1} = i|\psi_{t-1}] = \Pr[S_t = j|S_{t-1} = i] \times \Pr[S_{t-1} = i|\psi_{t-1}] \]

where \( \Pr[S_t = j|S_{t-1} = i] \) is the transition probability. The terms in the numerator are now in known terms. The denominator, \( \Pr[y_t|\psi_{t-1}] \), can be expressed as

\[ \Pr[y_t|\psi_{t-1}] = \sum_{j=0}^{1} \sum_{i=0}^{1} \Pr[y_t, S_t = j, S_{t-1} = i|\psi_{t-1}]. \]

Finally, summing over state \( i \) we get our single state posterior

\[ \Pr[S_t = j|\psi_t] = \sum_{i=0}^{1} \Pr[S_t = j, S_{t-1} = i|\psi_t]. \quad (25) \]
From the filter we obtain the density of \( y_t \) conditional on past information \( \psi_{t-1} \), \( t = 1,2,\ldots,T \). We can now calculate maximum likelihood estimates from the approximate log likelihood function

\[
\ell(\theta) = \ln[f(y_1, y_2, \ldots, y_T)] = \sum_{t=1}^{T} \ln(Pr[y_t | \psi_{t-1}]). \tag{26}
\]

Because these are switching models, the parameter vector set for both are going to have more parameters estimated than the non-switching model: \( \theta_{DNS-MSL} = \{A_{ij}, \Sigma_{\epsilon_{ij}}, \Sigma_{\eta_{ij}}, \mu, \lambda_0, \lambda_1\} \) and \( \theta_{DNS-MSV} = \{A_{ij}, \Sigma_{\epsilon_{ij}}, \Sigma_{\eta_{stij}}, \mu, \lambda_0\} \).

Once we have finished calculating the maximum of \( \ell(\theta) \), all parameters have been estimated and we can get inferences on \( S_t \) and \( f_t \) conditional on all the information in the sample: \( Pr[S_t = j | \psi_T] \) and \( f_t | T \) for \( t = 1,2,\ldots,T \). Instead of incrementing to the end of the sample as in the KF, to obtain smoothed probabilities and factors we increment from the end of the sample to the beginning, gathering all information along the way. So for \( t = T-1, T-2 \ldots, 1 \) we can approximate the smoothed joint probability

\[
Pr[S_t = j, S_{t+1} = k | \psi_T] \approx Pr[S_{t+1} = k | \psi_T] \times Pr[S_t = j | \psi_t] = \frac{Pr[S_{t+1} = k | \psi_T] \times Pr[S_t = j | \psi_T] \times Pr[S_t = j, S_{t+1} = k | \psi_T]}{Pr[S_{t+1} = k | \psi_T]} \tag{27}
\]

and probability

\[
Pr[S_t = j | \psi_T] = \sum_{k=0}^{1} Pr[S_t = j, S_{t+1} = k | \psi_T] \tag{28}
\]

These probabilities are used as weights in weighted averages to collapse the \((M \times M)\) elements of \( f^{(j,k)}_{t|T} \) and \( v^{(j,k)}_{t|T} \) into \( M \) where \( M = 2 \) for our model. These weighted averages over \( S_{t+1} \) are

\[
f^{j}_{t|T} = \frac{\sum_{k=0}^{1} Pr[S_t = j, S_{t+1} = k | \psi_T] f^{(j,k)}_{t|T}}{Pr[S_t = j | \psi_T]} \tag{29}
\]

and
\[ \nu_t^j = \frac{\sum_{k=0}^{1} \Pr[S_t = j, S_{t+1} = k | \psi_T] \left[ \nu_t^{(j,k)}(T) + \nu_t^{(j,k)}(T) \nu_t^{(j,k)}(T) \nu_t^{(j,k)}(T) \nu_t^{(j,k)}(T) \nu_t^{(j,k)}(T) \right]}{\Pr[S_t = k | \psi_T]} \] (30)

Taking a weighted average over the states at time \( t \) we get an expression for the smoothed factors

\[ f_{t|T} = \sum_{j=0}^{1} \Pr[S_t = j | \psi_T] f_{t|T}^j. \] (31)

This completes the KA. Further details and justifications can be found in Kim and Nelson (1999).

2.3 Data

We use end-of-month, bid-ask averages for U.S. Treasury yields from January 1970 through December 2000. Diebold and Li (2006) convert the unsmoothed Fama-Bliss (1987) forward rates given by CRSP to unsmoothed Fama-Bliss zero rates for the following eighteen maturities: 1, 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 month. The data is kindly supplied by Francis Diebold. Figure 2.1 gives a 3-dimensional mesh plot of the term structure for the sample period and the maturities. There are two primary advantages for us to limit our study to this particular sample period: first, this Diebold’s dataset is produced through consistent and careful cleaning procedures that remove microstructure noises and therefore provides us with a nice dataset to evaluate the proposed models; second, the most recent periods have featured an environment of extremely low nominal interest rates with the so-called zero lower bound constraint, which implies that the Gaussian type of term structure models typically encountered in the literature including the one implemented here would provide a poor approximation and thus calls for separate treatment that we leave for future studies.

Table 2.1 reports the means, standard deviations, and autocorrelations across maturities for the yields with maturities of 1, 12, and 30 months. The summary statistics show the average yield curve is upward sloping—a reflection of the risk premium inherent in longer maturities.
The volatility is generally decreasing by maturity with the exceptions of the one-month being less volatile than the 3, 6, and 9-month bills and the 8-yr being less volatile than the 9-yr bond.

We also report the statistics for the empirical counterparts for the level, slope, and curvature factors. It is worth declaring which convention we adopt in calculating the empirical factors. The empirical level factor is calculated as an average of the 1, 24, and 120 month maturities. The empirical slope factor is the difference between the 120 and 1 month maturities. Lastly, the empirical curvature factor is twice the 24 month maturity minus the sum of the 1 and 120 month.

2.4 Empirical Results

In this section we present the model estimation, comparative test results, and explore a macro-factor linkage with the DNS, DNS-MSL, and DNS-MSV models. Parameter estimates of each model via KF and KA are presented and discussed. We then look at in-sample estimation through root mean squared error analysis and information criterion calculations. We test to see if the Markov-switching models are significantly different from the baseline model using a bootstrapped LR test. Finally, we use logit regressions to explore if the estimated regimes are related to macro-factors for inflation expectations, economic activity and monetary policy.

2.4.1 DNS Model

The baseline DNS model is estimated with parameter estimate values close to those of other DNS parameter estimates in the literature. The parameter estimates are listed in Table 2.2. The estimate for the loading parameter $\lambda$ is 0.080 with a standard error of 0.0035 while the estimated $\lambda$ for Diebold, Rudebusch and Aruob (2006) is 0.077 and 0.078 for Koopman et al. (2010). Using the Diebold and Li’s interpretation of $\lambda$ we are able to ascertain the maturity in

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7 Some authors define empirical level as simply the observed long term maturity, which in our case would be $y(120)$. 

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which the loading on the curvature factor attains a maximum, henceforth referred to as implied maturities. Recall the loading on the curvature factor (CL) has functional form

\[
CL = \frac{1-e^{-\lambda m}}{\lambda m} - e^{-\lambda m}.
\]

Taking the first order condition of CL with respect to the maturity, \(m\), yields

\[
CL_m = \frac{e^{-\lambda m}}{\lambda} + me^{-\lambda m} - \frac{1-e^{-\lambda m}}{m\lambda^2}.
\]

Setting this nonlinear equation to zero and solving for \(m\) gives the maturity that the curvature loading reaches a maximum. The implied maturity of our estimated DNS model is 22.4 months while the implied maturity in the DRA paper is 23.3 months.

Our estimated smoothed factors from the DNS model have relatively high correlations to the empirical factors as shown in Table 2.3. In fact, the smoothed factors of this model give the highest correlations for each empirical factor of all the models estimated. Figure 2.2 gives a visual of the time-series of each estimated factor plot with the time-series of the empirical factors.

2.4.2 Loading Parameter Switching Model

We now introduce the first of our two regime switching models--the DNS-MSL model. Comparing the correlations of our estimated smoothed factors for this model and their respective empirical factors from Table 2.3 we find a drop in the correlations across the factors as compared with the baseline non-switching model. Specifically, the curvature factor experiences the largest decrease of all the models. This is further supported with a visual inspection of the third plot of the smoothed-empirical factor plots of Figure 2.3.

There are two causes for this drastic decrease in the correlation of the smoothed curvature factor and its empirical counterpart in both switching models. First, the empirical factors are calculated under the assumption of no switching in yields. Therefore the factors responsible for
capturing the switching we propose exists in the term structure—slope and curvature—should experience the largest decreases in correlation with empirical slope and curvature. Second, and more specifically for the curvature factor, the literature has shown that the curvature factor is highly volatile and thus may suffer from weak identification and therefore its estimation is the most tenuous of all the estimable factors. It is indeed the case that for each model we estimate the curvature factor has the highest volatility.

Our estimation results from the KF indicate the loading parameter $\lambda$ is subject to a hidden Markov switching component. We estimate $\lambda$ to be 0.055 (32.6 months) and 0.153 (11.7 months) for the low and high regimes, respectively. Figure 2.5 shows the effect these values have on the slope and curvature factor loadings across maturities.

In the first two plots we see a much faster decay of the slope and curvature loading factors in the high interest rate regime than in the low interest rate regime, and therefore the slope and curvature factor loadings influence medium and long-term maturities less than in the low regime. Relating this finding to the macro-factors literature discussed in Section 1, we say yields for medium and long-term maturities are determined more by long-term inflation expectations during regimes when $\lambda$ is relatively high and more by economic activity and monetary policy when $\lambda$ is low.

From the third plot in Figure 2.5, we see that the slope loading factor is uniformly greater across maturities in the low regime than in the high regime. This shows that the slope factor—proxying for monetary policy—contributes more to yield determination over all maturities in the low $\lambda$ regime relative to inflation expectations. In the last plot, the curvature loading factor is greater in the high regime than in the low regime for the one through 19-month maturities and therefore influences the yields of those maturities more so than in the low regime. For longer
maturities in the high regime, the curvature loading factor decays quickly and is less of a factor in yield determination than in the low $\lambda$ regime.

Figure 2.6 plots the time series of the unobserved state, $S_t$, of the DNS-MSL model. The timing and duration of the regime changes in Figure 2.6 coincide very nicely with periods of changing inflation expectations, economic activity and monetary policy. From Figure 2.6 we see in the 1970s and early 1980s, the high $\lambda$ regime was persistent. Based on our above discussions, these regimes correspond to periods in which inflation expectations weighed most heavily on Treasury yield determination. After the monetary policy experiment of the late 70s and early 80s, we witness more persistent periods where monetary policy plays a more important role in the bonds market which coincides with the Great Moderation. The peaks associated with a high $\lambda$ regime are less frequent and less persistent during this period from the late 1980s throughout the 1990s. Gurkaynak and Wright (2012) interpret the 1990s as a period where inflation uncertainty was diminishing. In Figure 2.6 we observe that the years from mid-1990 until mid-1994 are clearly categorized as a low $\lambda$ regime where monetary policy had a stronger effect on the bonds market. This period corresponds to the mild recession caused by the oil supply shock as a result of the Gulf War and the Federal Reserve’s lowering of the federal funds rate to stimulate economic activity.

2.4.3 Factor Volatility Switching Model

The second of our regime-switching models is the DNS-MSV model. As mentioned previously, authors investigating regime-switching in interest rates have consistently applied a hidden switching component to the conditional mean and volatility, simultaneously. After running pre-tests for Markov switching in the various model parameters, we find no significant switching in the conditional mean but we do find significant switching in the volatility.
parameters of each of the latent factors. The estimated factors display a decrease in their correlations with the empirical factors as compared to the correlations between the baseline model’s factors and empirical factors. But the decrease is not as great as that of the DNS-MSL correlations. And once again, the curvature factor experiences the smallest correlation with its empirical counterpart. This is visibly evident in the third plot of Figure 2.4.

From Table 2.2 we find the factor volatilities for the DNS-MSV model increase from 0.26, 0.33, and 0.66 in the low volatility regime to 0.50, 1.21, and 1.87, respectively, in the high volatility regime. These are relative increases of 92 percent, 267 percent, and 183 percent, respectively. We estimate \( \lambda \) to be 0.081. Our \( \lambda \) estimate gives an implied maturity of 22.1 months which is slightly smaller than the baseline’s implied maturity of 22.4 months. The latent factor, \( S_t \), is depicted pictorially in Figure 2.7. The smoothed probability plot shows ephemeral and sustained periods of high factor volatility in the 1970s. This is more than likely a result of increased inflation uncertainty due to the oil supply shocks. Cosimano and Jansen (1988) suggest that the increased conditional variance of energy prices is ultimately the culprit. A sustained high factor volatility regime is estimated from around 1980 until 1983 which covers the monetary policy experiment period where the Federal Reserve began targeting money aggregates in late 1979 and then switched back to targeting the Federal Funds Rate in late 1982. These changes in monetary policy manifested itself as increased uncertainty in US financial markets during this period. Factor volatilities began to show a sustained decrease in the mid-1980s with ephemeral spikes occurring until 1988 where afterwards the volatilities are in the low regime until the end of the sample period. The timing of this decrease corresponds to the literature’s exploration of the Great Moderation. Kim and Nelson (1999), Stock and Watson (2003), and Fogli and Perri (2006) all find evidence of a decrease in the volatility of US economic activity around 1984,
which is close to when the factor volatilities began to show a transition to the long-term low regime.

In accordance with the literature, we estimate a general model where both the decay parameter and factor disturbances are subject to switching, simultaneously. The RMSE results show this model to be superior to the DNS-MSV in-sample fit results but inferior to the DNS-MSL results. We do not include the results of this most general model in this paper but are available upon request.

2.4.4 In-Sample Forecasting

Table 2.4 reports our in-sample root mean squared error (RMSE) values for the various models. Recall that the KF estimates measurement error parameters for each maturity. These parameter estimates are recorded as the diagonal terms of the covariance matrix of the measurement equation error. Taking these diagonal elements we are able to calculate the RMSE according to the formula

$$\left(\sum_{1}^{T}(y_t - \hat{y}_t)^2 / T\right)^{1/2}$$

where $T = 371$ for all models. We find overall the DNS model yields the largest average RMSE and the DNS-MSL yields the smallest average RMSE. In terms of percentage changes, the DNS-MSL and DNS-MSV models decrease the in-sample average RMSE by 2.86 percent and 0.37 percent, respectively.

In addition to calculating the total average RMSE for all maturities, we calculate average RMSEs for the ranges of short, medium, and long-termed maturities. We limit the three maturity ranges to 6 maturities, i.e. short maturity range contains the 1, 3, 6, 9, 12 and 15m maturities, etc. The DNS-MSL decreases the average RMSEs for the short, medium, and long maturity range groups by 4.89 percent 2.47 percent, and 0.75 percent, respectively, the largest decreases for the
respective groups. The improved modeling of the DNS-MSV model came with a decreased average RMSE for the short maturity range group by 1.30 percent but for the medium and long groups the average RMSE actually increased by 0.30 percent and 0.17 percent respectively over the DNS model. The DNS-MSV model is capturing the volatility of the short maturity bills but the bonds have less volatility so the model is over-identified for longer maturities resulting in greater loss of estimation efficiencies. These results support the case for applying a switching component to the DNS model. Furthermore, these results suggest switching $\lambda$ leads to a noticeable improvement for the in-sample fit over the DNS model by better estimating longer termed maturities, a known deficiency of the DNS model.

The maximum log-likelihood value and AIC/BIC measures are calculated for each model and are given in Table 2.2. Our regime switching models show significant model improvement over the baseline DNS model. The DNS-MSL model achieves the greatest log-likelihood value and the smallest AIC and BIC values, further strengthening the conclusion that this model performs the best in-sample fit of all the models.

2.4.5 Testing for Statistical Difference in Models

We have established the DNS-MSL and DNS-MSV models outperform the DNS model with in-sample estimation. We now show the two models are statistically different from the DNS model. Since both the DNS-MSL and DNS-MSV models nest the DNS model, the likelihood ratio (LR) test is a good statistical testing candidate to address our issue. But we are unable to make accurate statistical inference using the asymptotic $LR$ distribution for two reasons. First, the finite sample size may render the asymptotic theory less accurate in practice. Second, and more importantly, we encounter a nuisance parameter problem which renders asymptotic distributions of the $LR$ test nonstandard. This issue has been addressed by Davies (1977),
We follow Hansen’s simulation methodology to design a bootstrap procedure using the $LR$ test.\footnote{Our regime switching methodology is more closely related with Hansen (1992) when testing in the presence of nuisance parameters than Andrews and Ploberger (1992) which apply their test in the context of a single break in dynamics.}

We outline the steps used to bootstrap the $LR$ distribution for the comparison of the DNS and DNS-MSL models:

**STEP 1:** Obtain the max likelihood value ($LLV_0$) of the DNS model (null) and the max likelihood value ($LLV_A$) of the DNS-MSL model (alternative), using the real dataset. Calculate the $LR$ statistic.

**STEP 2:** Generate yields ($\hat{y}_0$) using the DNS model. Fit the DNS-MSL and DNS models to the generated yields ($\hat{y}_0$). Obtain the max likelihood value ($LLV^*_A$) for the DNS-MSL model and ($LLV^*_0$) for the DNS model. Calculate the new $\hat{LR}_j^*$ using $LLV^*_0$ and $LLV^*_A$.

**STEP 3:** Using $\hat{LR}$ and $\hat{LR}_j^*$ compute a bootstrap critical value ($\hat{C}^*_\alpha$). For a test at level $\alpha$, first sort the $\hat{LR}_j^*$ from smallest to largest. Then calculate

$$\hat{C}^*_\alpha = \hat{LR}_\alpha^{(B+1)}$$

where $\alpha$ represents your confidence level and $B$ is the number of bootstraps.

Repeat steps 2 and 3 $B$ times and obtain $\{\hat{LR}_j^*\}_1^B$.

**STEP 4:** Reject the null hypothesis if $\hat{LR} > \hat{C}^*_\alpha$.

The steps are the same for deriving the $LR$ distribution for the DNS and DNS-MSV comparison. The $LR$ test statistic under the null of no switching is 467 for the DNS-MSL model and 410 for the DNS-MSV model. We perform 1000 bootstraps to derive a $LR$ distribution. Figure 2.8 is a plot of the probability density for both models using a normal kernel function to smooth. Table
2.5 lists the critical values for each model at the 10%, 5% and 1% confidence levels. It is evident that the test statistic greatly exceeds all bootstrapped critical values so we are able to reject the null of the linear DNS model. This greatly enhances our stance that term structure modeling should take into account regime switching and that a model without regime switching is subject to omitted variable bias.

2.4.6 Logit Regression Analysis of Yields-Macro Link

A number of papers have tried to establish relationships between yield curve factors, such as the NS factors or the first three principal components of the term structure, with macro-factors. We start with a look at correlations between the level and slope factors of our various models with macro-factors inflation expectations and capacity utilization. Inflation expectations data comes from the University of Michigan survey in the St. Louis FRED database and covers the period January 1978 through December 2000. Capacity utilization which serves as our proxy for economic activity also comes from the FRED database and covers the period January 1970 through December 2000. Table 2.3 shows the correlations for the smoothed factors of the DNS, DNS-MSL, and DNS-MSV models and the macro-factors. Correlations between inflation expectations and the smoothed level factors of the various models are about 0.4. This is a good indicator that the level factor is capturing the dynamics of inflation from the late 1970s through 2000. The DNS model yields the highest correlation (0.4135) while the DNS-MSL model gives the smallest (0.3982). Correlations between smoothed slope factors and capacity utilization are about -0.2. The negative correlation here indicates that recessions are related with a larger magnitude of the slope factor and hence steeper curve. This tells us that the slope factor is more related with the counter-cyclical monetary policy. In recessions the Fed tends to lower the short rate (recall the Taylor rule) and thus steepens the yield curve. Although flatter yield curve tends
to predict recessions, steeper curve is contemporaneously related with recessions. The DNS model gives the largest absolute value for the correlation (0.1995) while the DNS-MSL gives the smallest absolute value (0.1935).

The investigations of Bansal and Zhou (2002), Clarida et al. (2006), and Xiang and Zhu (2013) show a relationship between interest rate regimes and real economic activity. To investigate a similar relationship with our regime switching models, we estimate a logit model using the monthly capacity utilization data as our measure of economic activity and transform smoothed probabilities from the DNS-MSL and DNS-MSV models into a binary probability variable, \( p(y) \). The binary variable is defined to assume the value of zero when the smoothed probability value is greater than or equal to one-half and one when less than one-half. Thus, we focus on the low regimes. Figures 2.9 and 2.11 are smoothed probability plots for the regime switching models juxtaposed on a plot of the macro-factors. Figures 2.10 and 2.12 are transformed binary probability plots for the regime switching models juxtaposed on macro-factors. The logit model takes the following form

\[
p(y_{\text{low}}) = \frac{\exp(\beta_0 + \beta_1 \text{CAPUTIL})}{1 + \exp(\beta_0 + \beta_1 \text{CAPUTIL})}
\]

where \( p(y_{\text{low}}) \) is the transformed smoothed probability of being in the low interest rate level regime for the DNS-MSL model and the low volatility regime for the DNS-MSV model.

The sample period we investigate with the logit regressions is January 1985 through December 2000 to mitigate the interest rate volatility over the period associated with the monetary policy experiment of the late 1970s and early 1980s and to take advantage of the economic stability associated with the Great Moderation.

In times of low interest rate volatility, manufacturers will increase planned investment spending due to the low level of economic uncertainty. Thus we would expect increased
economic activity. The DNS-MSV logit model should reflect this relationship with positive coefficients for capacity utilization when regressing on transformed low volatility regime smoothed probabilities. The DNS-MSV logit model gives $\hat{\beta}_1 = 1.33 (0.59)$ which implies the odds of being in the low volatility regime increases by 74%. The Mcfadden pseudo-$R^2$ of this regression was 0.31. Xiang and Zhu (2013) also find a significantly positive coefficient for their logit model when the low volatility regime probabilities are used. This result supports the economic prior that being in a low volatility regime is more likely to occur during an economic expansion.

The DNS-MSL logit parameter estimates yields interesting results that suggest an asymmetric impact of monetary policy on the yield curve over the business cycles. The logit regression yields a parameter estimate of $\hat{\beta}_1 = -0.58 (0.11)$ with a pseudo-$R^2$ of 0.15. This estimate translates to a 79% decrease in the odds of being in the low lambda regime. In other words, increasing capacity utilization or an economic boom tends to be associated with the high $\lambda$ regime, in which the slope factor has a relatively smaller impact on the yields curve compared to the low $\lambda$ regime (recall the bottom left plot of Figure 5). Since the slope factor normally approximates the monetary policy this finding suggests that the monetary policy seems to have a larger impact on the yields curve during recessions than expansions. This reveals an interesting asymmetric effect of the monetary policy on the economy through the yields curve.

2.5 Conclusion

In this paper we investigate and model the parameter instability in the term structure using regime-switching dynamic Nelson-Siegel models. After applying a hidden Markov switching component to all of the model’s parameters one at a time, we find that the factor

---

9 For the logit regression a sufficient condition for a satisfactorily good model fit is when the McFadden pseudo-$R^2$ falls within the interval $[0.2, 0.4]$. 

loading parameter and the factors’ conditional volatilities show significant switching when allowed—not the conditional mean as noted in the literature. Specifically, the model allowing switching loading parameters yields smaller AIC/BIC values and produces smaller root mean squared error values for most of the individual maturities. The model also produced smaller RMSEs across maturity groupings, and a smaller total RMSE. Overall this model gives a more accurate timing of regime duration in the term structure over the sample period. We also test to see if both models are statistically different from the non-switching model using a LR test. To correct for non-standard errors due to a nuisance parameter issue, we bootstrap critical values for the test. Our testing results show that both models are statistically different from the non-switching model at the one percent confidence level, thus supporting an extension of the DNS model to include regime switching components.

Finally, we find that the extracted factors from our DNS models are closely related with the macro-economy. In particular, the level factor appears to be strongly correlated with inflation expectations while the slope factor seems to be counter-cyclical, which is consistent with some previous findings, such as Wu (2002), that the slope factor may well be closely related with monetary policy. In addition, we also find that the regime switching in the extended DNS models coincides with economic activity and monetary policy changes. The model accounting for regime changes in volatility captured the timing of volatility regimes associated with the oil price shock of the 1970s, the monetary policy experiment of the early 1980s and the period known as the Great Moderation. The model accounting for regime switching in the loading parameter suggests an interesting asymmetric effect of monetary policy on the yield curve. Specifically, we find that the monetary policy tends to have a larger impact on the yield curve during recessions than expansions.
2.6 References


Davies, R. B. (1977). Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika, 64*(2), 247-254.

----------------- (1987). Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika, 74*(1), 33-43


Table 2.1
Descriptive Statistics

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Empirical Factors

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<th>Slope</th>
<th>Curvature</th>
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<td>4.44</td>
<td>14.93</td>
<td>0.43</td>
<td>0.84</td>
<td>-1.80</td>
</tr>
</tbody>
</table>

Note: We define the Level as $(y(1)+y(24)+y(120))/3$, the Slope as $y(120) - y(1)$ and Curvature as $2y(24) - (y(1) + y(120))$. 
Table 2.2
Parameter Estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DNS</th>
<th>DNS-MSL</th>
<th>DNS-MSV</th>
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<tr>
<td>$a_{11}$</td>
<td>0.95</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(0.0178)</td>
<td>(0.0045)</td>
<td>(0.0054)</td>
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<td>$a_{22}$</td>
<td>0.91</td>
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<tr>
<td></td>
<td>(0.0256)</td>
<td>(0.0150)</td>
<td>(0.0094)</td>
</tr>
<tr>
<td>$a_{33}$</td>
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<td>0.79</td>
<td>0.90</td>
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<tr>
<td></td>
<td>(0.0569)</td>
<td>(0.0322)</td>
<td>(0.0244)</td>
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<tr>
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<tr>
<td></td>
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<td>(0.3913)</td>
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<td>-0.66</td>
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<td></td>
<td>(0.0543)</td>
<td>(0.2409)</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td>(0.0283)</td>
<td>(0.0521)</td>
<td>(0.0438)</td>
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<td>$\sigma_{\eta L}^1$</td>
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<td>-----</td>
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<td></td>
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<tr>
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<tr>
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<td>$\lambda^0$</td>
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<td>0.081[22.1]</td>
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<td>(0.0017)</td>
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<td>Max. Likelihood Value</td>
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<td>9481.7</td>
<td>9435.3</td>
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Number of free parameters ($k$) | 28 | 31 | 33 |
AIC | -18431.2 | -18901.4 | -18804.6 |
BIC | -18240.6 | -18690.4 | -18578.0 |

Standard errors are in parentheses

Implied maturities measured in months are in brackets

We calculate the information criteria according to the formulas $AIC = -2 \times \ell(\theta)_{\text{max}} + 2 \times k$ and $BIC = -2 \times \ell(\theta)_{\text{max}} + k \times \ln(NT)$, where $N = 18$ maturities and $T = 371$ months
Table 2.3
Correlations

<table>
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<tr>
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<th>DNS-MSL</th>
<th>DNS-MSV</th>
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<td>$\rho(L_{EMP}, L_{EST})$</td>
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<td>0.9061</td>
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<tr>
<td>$\rho(C_{EMP}, C_{EST})$</td>
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<td>0.6374</td>
<td>0.7429</td>
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<tr>
<td>$\rho(L_{EST}, \text{INFEXP})$</td>
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<td>$\rho(S_{EST}, \text{CAPUTIL})$</td>
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<td>-0.1984</td>
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Table 2.4
In-Sample Forecast: Root Mean Squared Errors

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<th>DNS-MSV RMSE</th>
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<tr>
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<td>0.3255</td>
<td>0.3101</td>
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<tr>
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<tr>
<td>120</td>
<td>0.4122 (bold)</td>
<td>0.4140</td>
<td>0.4145</td>
</tr>
<tr>
<td>Average</td>
<td>0.3453 (bold)</td>
<td>0.3354</td>
<td>0.3440</td>
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</table>

1-15mo    | 0.4038   | 0.3838       | 0.3985       |
18-48mo   | 0.2851   | 0.2781       | 0.2860       |
60-120mo  | 0.3469   | 0.3443       | 0.3475       |
Table 2.5
Likelihood Ratio Critical Values (Bootstrapped)*

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<td>55.18</td>
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<td>DNS-MSV</td>
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<td>37.06</td>
<td>55.90</td>
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</table>

* 1000 Bootstraps were performed to estimate critical values for each model
Figure 2.1: US Term Structure
Figure 2.2: DNS Smoothed Factors
Figure 2.3: DNS-MSL Smoothed Factors
Figure 2.4: DNS-MSV Smoothed Factors
Figure 2.5: Regime Plot for Factor Loadings
Figure 2.6: DNS-MSL: Smoothed Probability Plot

Figure 2.7: DNS-MSV Smoothed Probability Plot
Figure 2.8: Bootstrapped Likelihood Ratio Test Distribution
Figure 2.9: Smoothed Probability (DNS-MSL) and Macro-Fundamentals

Figure 2.10: Logit Probability (DNS-MSL) and Macro-Fundamentals
Figure 2.11: Smoothed Probability (DNS-MSV) and Macro-Fundamentals

Figure 2.12: Logit Probability (DNS-MSV) and Macro-Fundamentals

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3.1 Introduction

At first glance, the interaction between the UK macroeconomy and the term structure of interest rates appears to mirror that of the US. Both countries experienced periods of sustained high inflation in the post-WWII period which had deleterious effects on output and consumer economic outlook. In response, the US and UK initially focused their monetary policy initiatives on monetary aggregates. When that did not create the desired effect, both countries targeted inflation through the manipulation of their respective monetary policy rates. This in turn established what came to be known as a “Great Moderation” where the level and volatility of not only inflation but many other macroeconomic variables decreased ushering an era of unprecedented economic growth for both nations.

Just as in the US, the cause of the UK’s great moderation has been greatly debated between those who believe in good policy implementation in the form of inflation targeting after UK’s exit from the Exchange Rate Mechanism in September 1992 and those who take a stand with good luck playing the dominant role in a more stable macroeconomic environment. King (1997) attributes the UK great moderation to a combination of explicit inflation targeting and institutional measures working in tandem to promote policy transparency to market participants. Chadha and Nolan (2001) find that increased information flow to participants in the form of published inflation forecasts and announcements in policy rate changes has been associated with a less volatile interest rate regime. Benati (2008) using a Bayesian time-varying parameters
structural VAR with stochastic volatility finds evidence in support of good luck from counterfactual simulations. The results of this paper indicate that imposing the post-May 1997 monetary rule on the period 1959Q4 – 2005Q4 would have minimal impact on money growth, inflation, output growth and interest rates. Although the aim of this paper is not to weigh in on the good policy vs. good luck debate, it is important to discuss the importance surrounding 1992 for the purposes of showing the increased linkage between the UK term structure and macroeconomic fundamentals.

In the literature, the link between the term structure and the macroeconomy is established through relating macroeconomic fundamentals to yield curve factors. Litterman and Scheinkman (1991) denoted the latent yield curve factors “level”, “slope”, and “curvature” and showed they accounted for a majority of a yield curve’s shape and movement. There is strong evidence that relates the level yield curve factor with market participants’ inflation expectations as shown by Stock and Watson (2005) and Dewachter and Lyrio (2006) for the US economy. A similar result has been established by Bianchi et al. (2009) among others for the UK. Chen (1991), Estrella et al. (2003), and Diebold et al. (2006) find a link between the slope factor and real economic activity. The curvature factor has been a bit harder to relate to a macroeconomic fundamental variable. Because of the curvature factor’s high correlation to the slope factor it has been related to macroeconomic fundamentals linked to the slope factor. Using a no-arbitrage affine term structure model within a New Keynesian macro framework, Bekaert et al. (2009) find that monetary policy shocks account for much of the variation in the slope and curvature factors.

Although there have been linkages made between yield curve factors and macroeconomic fundamentals as discussed above, factors and macro-variables remain imperfectly correlated. A growing literature has developed to address these unspanned macro-risks in modeling the term
structure which would give financial advantages to market participants and economic insight to policy makers. These models add observable macro-factors such as inflation, real economic activity, and monetary policy to latent yield curve factors in estimating the term structure. Wu (2002), Ang and Piazzesi (2003), and Joslin et al. (2010), utilize principal components analysis to decompose the term structure into three orthogonal latent yield curve factors within a no-arbitrage affine term structure model (ATSM) framework. Diebold et al. (2006, henceforth DRA) and Bianchi et al. (2009, henceforth BMS) employ the dynamic factor model framework of Diebold and Li (2006) to obtain yield curve factors. It is this latter strand of literature that this paper is most closely related.

Although integrating macroeconomic fundamentals into term structure estimation does alleviate the issue of unspanned macro-risks, it does not address the noted parameter instability associated with the term structure. For the US, this issue has been investigated in the works of Cogley (2004) and Rudebusch and Wu (2007) and for the UK, Benati (2004) and BMS. BMS models parameter instability with a time-varying coefficient and stochastic volatility VAR augmented with yield curve factors. Ang and Bekaert (2002) model the term structures of the US, UK and Germany within a regime switching framework on the belief that regimes in expected inflation or business cycles are responsible for regimes in nominal interest rates. They find regime classification for the UK is weakly identified with high frequency of regime switching. Levant (2014) utilizes a number of structural break tests and finds dis-inflationary policies impacted different ends of the yield curve for both countries. In the US, significant structural breaks were found in the longer maturities of the term structure, while in the UK breaks were detected in the shorter maturities when monetary policy switched to inflation
targeting. This result suggests the interaction between the macroeconomy and the bonds market differs for countries despite similar goals and implementation of monetary policy.

This paper presents strong evidence that the interaction between the UK term structure and macroeconomy has changed significantly with the inception of inflation-targeting post-October 1992. Employing a macro-factor augmented dynamic Nelson-Siegel model, yield curve factors are related to macroeconomic fundamentals during the current monetary policy regime for the UK. In particular, the results of this paper relate slope factor dynamics to monetary policy and curvature dynamics to real economic activity during the inflation-targeting era. We see an improvement in the ability of the level factor to capture inflation expectations through persistent increases in real economic activity and inflation expectations in the presence of a level shock post-1992, in contrast to pre-1992 where the level shock had a negligible effect on real economic activity and inflation expectations. Finally, the long-run effects of a monetary policy shock to the level factor changed from positive pre-1992 to negative post-1992, a result corroborated by the changing of inflation expectations from positive values before inflation-targeting to negative values during the inflation-targeting regime in the presence of a monetary policy shock. I also find a bi-directional interaction between the slope yield curve factor and the macroeconomy for the pre-1992 era but post-1992 the bi-directional interaction changes to the level and curvature yield curve factors and the macroeconomy.

Having established linkages between yield curve factors and macroeconomic fundamentals, I employ Markov-switching dynamic Nelson-Siegel models to not only model parameter instability in the UK term structure but also relate estimated regimes to macroeconomic fundamentals. The results of this paper indicate a decrease in term structure volatility post-1992 with more frequent and persistent low volatility regimes. These regimes
correspond to periods where monetary policy and economic activity have a greater influence on the bonds market given how yield curve factors relate to macroeconomic fundamentals.

The paper is organized as follows. Section 2 presents generalizations of the Nelson-Siegel (1987) model of two varieties: (1) a first-order factor augmented vector auto-regression (FAVAR) within a state-space framework and (2) a two-state Markov-switching VAR within a state-space framework. Section 3 describes and presents the term structure and macroeconomic fundamentals data. Section 4 presents the empirical results and Section 5 concludes.

3.2 Models

3.2.1 Dynamic Nelson-Siegel Model \((state-space framework)\)

Diebold, Rudebusch, and Aruoba (2006, henceforth DRA) estimates the Diebold and Li (2006, henceforth DL) factorization of the NS yield curve model according to a state-space framework. The advantage of this approach as compared to the two-step approach used in the DL paper is that estimation of the latent NS yield curve factors and the yields are done in one step. Factor estimation uncertainty is ignored in the two-step approach, but is accounted for in the state-space approach which yields correct inference via standard asymptotic theory given the estimated parameters. The measurement equation for the DRA state-space framework for the DL factorization is

\[
\begin{bmatrix}
1 - e^{-\lambda m_1} \\
\frac{\lambda m_1}{1 - e^{-\lambda m_1}} \\
\vdots \\
\frac{\lambda m_N}{1 - e^{-\lambda m_N}}
\end{bmatrix}
\begin{bmatrix}
\beta_{1t} \\
\beta_{2t} \\
\vdots \\
\beta_{3t}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{t}(m_1) \\
\varepsilon_{t}(m_2) \\
\vdots \\
\varepsilon_{t}(m_N)
\end{bmatrix}
\]

\(1\)

or written in matrix notation as

\[
y_t(m) = \Lambda(\lambda)F_t + \varepsilon_t, \quad \varepsilon_t \sim MN(0, G), t = 1, \ldots, T, \quad (2)\]
where \( y_t(m) \) is the vector of yields as functions of maturity, \( F_t = [\beta_{1t}, \beta_{2t}, \beta_{3t}]' \) where \( \beta_{1t}, \beta_{2t}, \) and \( \beta_{3t} \) represent the time-series of the level, slope, and curvature yield curve factors, respectively. DRA allow restrict the measurement error variance-covariance, \( G \), to be diagonal which is a standard assumption for yield models. This equation models the dynamics of the term structure from latent factors and a particular loading structure on each factor. The transition equation is represented by

\[
\begin{bmatrix}
\beta_{1t} - \mu_{L_t} \\
\beta_{2t} - \mu_{S_t} \\
\beta_{3t} - \mu_{C_t}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\beta_{1t-1} - \mu_{L_{t-1}} \\
\beta_{2t-1} - \mu_{S_{t-1}} \\
\beta_{3t-1} - \mu_{C_{t-1}}
\end{bmatrix} +
\begin{bmatrix}
\eta_{L_t} \\
\eta_{S_t} \\
\eta_{C_t}
\end{bmatrix}
\]

(3)

or written in matrix notation as

\[
(F_t - \mu) = A(F_{t-1} - \mu) + \eta_t, \quad \eta_t \sim MN(0, Q), t = 1, ..., T
\]

(4)

which can be equivalently written as

\[
F_t = (I - A)\mu + AF_{t-1} + \eta_t,
\]

(5)

for estimation purposes which will be discussed later. DRA try two identification schemes of the state-space framework where the variance-covariance matrix, \( Q \), is allowed to be non-diagonal, thus, permitting shocks between factors and restricted to be diagonal, imposing no shocks between factors. They find the difference in the parameter estimates between these identification schemes is negligible with and without macro-factors added to the model.

3.2.2 Macro-Factor Augmented Dynamic Nelson-Siegel Model

Similarly to DRA, I add macro-factors to their state-space framework in order to investigate the relationship between the NS factors and macroeconomic fundamentals. The United Kingdom data included are related to a monetary-policy interest rate \( (R_t) \), total industrial production \( (IP_t) \), inflation expectations \( (IE_t) \). The choice for these particular macro-factors from
what is widely considered to be the minimum set of macroeconomic variables required to model fundamental macroeconomic dynamics as argued by Rudebusch and Svensson (1999) and Kozicki and Tinsley (2001). Following the DRA set up, the measurement equation for this investigation is formulated as

\[
\begin{bmatrix}
 y_t^1 \\
 y_t^2 \\
 \vdots \\
 y_t^N \\
 R_t \\
 IP_t \\
 IE_t
\end{bmatrix} =
\begin{bmatrix}
 1 & \frac{1-e^{-\lambda m_1}}{\lambda m_1} & \frac{1-e^{-\lambda m_1}}{\lambda m_1} & -e^{-\lambda m_1} & 0 & 0 \\
 1 & \frac{1-e^{-\lambda m_2}}{\lambda m_2} & \frac{1-e^{-\lambda m_2}}{\lambda m_2} & -e^{-\lambda m_2} & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 1 & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & \frac{1-e^{-\lambda m_N}}{\lambda m_N} & -e^{-\lambda m_N} & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
 L_t \\
 S_t \\
 C_t \\
 R_t \\
 IP_t \\
 IE_t
\end{bmatrix}
+ \begin{bmatrix}
 \varepsilon_t^1 \\
 \varepsilon_t^2 \\
 \vdots \\
 \varepsilon_t^N
\end{bmatrix}
\tag{6}
\]

or written in matrix notation as

\[
y_t^m = \Lambda(\lambda) F_t^A + \varepsilon_t, \quad \varepsilon_t \sim MN(0, G), \ t = 1, ..., T, \tag{7}\]

where \( F_t^A = [L_t, S_t, C_t, R_t, IP_t, IE_t]' \). G is assumed to be diagonal as in DRA. To ensure the macro-factors do not contribute to the estimation of the term structure when using the KF, the loading factors contribution of the macro-factors are restricted to zero. Also, the goal of this formulation is to not predict the macro-factors so to ensure this the loading factors are set at one and the errors associated with each macro-factor are set to zero. The transition equation is as follows

\[
\begin{bmatrix}
 L_t - \mu_Lt \\
 S_t - \muLt \\
 C_t - \muLt \\
 R_t - \muLt \\
 IP_t - \muLt \\
 IE_t - \muLt
\end{bmatrix} =
\begin{bmatrix}
 a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
 a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\
 a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\
 a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\
 a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\
 a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66}
\end{bmatrix}
\begin{bmatrix}
 L_{t-1} - \mu_Lt \\
 S_{t-1} - \muLt \\
 C_{t-1} - \muLt \\
 R_{t-1} - \muLt \\
 IP_{t-1} - \muLt \\
 IE_{t-1} - \muLt
\end{bmatrix}
+ \begin{bmatrix}
 \eta_{Lt} \\
 \eta_{St} \\
 \eta_{Ct} \\
 \eta_{Rt} \\
 \eta_{IPt} \\
 \eta_{IE_{t-1}}
\end{bmatrix}
\tag{8}
\]

or written in matrix notation as

\[
(F_t^A - \mu) = A(F_{t-1}^A - \mu) + \eta_t, \quad \eta_t \sim MN(0, Q), \ t = 1, ..., T \tag{9}
\]
where $F_t^A$ is the vector of parameters listed in the measurement. Equations 7 and 9 constitute what I will refer to as the macro-factor augmented dynamic Nelson-Siegel (MFA-DNS) model.

In this investigation, I assume $Q$ is diagonal for the sake of parsimony and computation time. In DRA, parameter estimates changed marginally when estimated under a diagonal $Q$ matrix or non-diagonal $Q$ matrix. This identification specification restricts interactions of the factors (latent and macro) only through the FAVAR(1) coefficients. Essentially, this translates to a factor’s dynamics being explained by how much all other factors contribute to its estimation and not through contributions from factor shocks. Therefore, the prediction of the latent factors within the KF scheme benefit from the additional information given by the unspanned macro-risks which in turn contributes to possibly better term structure estimation. This specification of the measurement and transition equations allow for estimation of impulse responses for the factors under consideration to better investigate the link between NS factors and UK macro-factors. It is worth noting this identification strategy is neutral with respect to ordering of the variables.

3.2.3 Impulse Responses

With the MFA-DNS model, it is possible to explore the interaction of the term structure and the macroeconomy through impulse responses. For the FAVAR that is Equation (9) it is possible to find a vector moving average representation which according to Granger and Diebold (1986) is

$$F_t^A = \mu + \sum_{j=0}^{\infty} \phi_j \varepsilon_{t-j}$$

(10)
where \( \phi^j = \begin{bmatrix} \phi_{11}^j & \cdots & \phi_{16}^j \\ \vdots & \ddots & \vdots \\ \phi_{61}^j & \cdots & \phi_{66}^j \end{bmatrix} \) are impulse response functions and \( \epsilon_t = \begin{bmatrix} \epsilon_{Lt} \\ \epsilon_{St} \\ \epsilon_{Ct} \\ \epsilon_{Rt} \\ \epsilon_{IPT} \\ \epsilon_{IEt} \end{bmatrix} \) are the innovations and have distribution \( \epsilon_t \sim N(0, \Sigma) \). Note this representation is simply a multivariate Wold decomposition of \( F_t^A \). To ensure the elements of \( \epsilon_t \) are not correlated, a Cholesky decomposition is applied to \( \text{Var}(\epsilon_t) = \Sigma \) so that \( \Sigma = CC' \) is lower diagonal and where

\[
C = \begin{bmatrix}
    c_{11} & 0 & 0 & 0 & 0 & 0 \\
    c_{21} & c_{22} & 0 & 0 & 0 & 0 \\
    c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\
    c_{41} & c_{42} & c_{43} & c_{44} & 0 & 0 \\
    c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & 0 \\
    c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66}
\end{bmatrix}
\]

with a non-zero diagonal. Given this new structure for \( \Sigma \), Eq. (10) is re-formulated as

\[
F_t^A = \mu + Cu_t + \sum_{j=0}^{\infty} \theta^j u_{t-j}
\]

where \( \theta^j = \phi^j C \) and \( u_{t-j} \equiv C^{-1} \epsilon_{t-j} \). Now the matrix sequence \( \theta^j \) is an orthogonalized impulse response function since

\[
\mathbb{E}(u'_t u_t) = C^{-1} \Sigma (C^{-1})' = C^{-1} CC' (C^{-1})' = I_6.
\]

3.2.4 Markov-Switching Dynamic Nelson-Siegel Models

Since the goal of this paper is to examine the macroeconomic fundamentals that contribute to regime changes in the UK term structure, I now introduce models developed by Levant and Ma (2014) that attempt to capture regime changes in the term structure by modeling changes in NS factor loadings and factor volatilities. The dynamic Nelson-Siegel Markov switching lambda (DNS-MSL) model is given by the following equations

\[
y_t = A(\lambda_S t) F_t + \epsilon_t, \quad \epsilon_t \sim MN(0, G), \ t = 1, \ldots, T,
\]

\[
(F_t - \mu) = A(F_{t-1} - \mu) + \eta_t, \quad \eta_t \sim MN(0, Q), \ t = 1, \ldots, T,
\]

\[
\lambda_S_t = \lambda_0 (1 - S_t) + \lambda_1 S_t, \quad S_t = 0, 1,
\]
where $\Lambda(\lambda_{S_t})$ models the factor loadings for the slope and curvature yield curve factors according to a two-state, $S_t$, Markov chain. This specification allows the loading parameter $\lambda$ to assume two values which permits the macroeconomic fundamentals that the slope and curvature factors are proxying for the freedom to change how they are weighted on yields according to the observed regime of the term structure under this model.11

The next model applies the hidden Markov-switching component to the factor volatilities. The dynamic Nelson-Siegel Markov switching volatility (DNS-MSV) model is

$$
\begin{align*}
\gamma_t &= \Lambda(\lambda) F_t + \varepsilon_t, \quad \varepsilon_t \sim MN(0, G), \ t = 1, \ldots, T, \\
(F_t - \mu) &= A(F_{t-1} - \mu) + \eta_{S_t}, \quad \eta_{S_t} \sim MN(0, Q(S_t)), \ t = 1, \ldots, T, \\
\eta_{S_t} &= \eta_0 (1 - S_t) + \eta_1 S_t, \quad S_t = 0, 1,
\end{align*}
$$

where $Q(S_t)$ is assumed to be diagonal and models the factor volatilities as a two-state Markov process. In the Nelson-Siegel framework yields are affine functions of the latent factors so if the factor volatilities experience switching this is tantamount to the yield volatilities being subjected to Markov switching.

Finally, I introduce a generalized version of the two previous models that links changes in the volatility structure of the term structure to macroeconomic fundamentals. The dynamic Nelson-Siegel Markov switching lambda-volatility (DNS-MSLV) model is governed by the following equations

$$
\begin{align*}
\gamma_t &= \Lambda(\lambda_{S_t}) F_t + \varepsilon_t, \quad \varepsilon_t \sim MN(0, G), \ t = 1, \ldots, T, \\
(F_t - \mu) &= A(F_{t-1} - \mu) + \eta_{S_t}, \quad \eta_{S_t} \sim MN(0, Q(S_t)), \ t = 1, \ldots, T, \\
\lambda_{S_t} &= \lambda_0 (1 - S_t) + \lambda_1 S_t, \quad S_t = 0, 1,
\end{align*}
$$

11 From the construction of the model, Levant and Ma (2014) show that low-lambda regimes facilitate monetary policy having a greater influence on yield determination while high-lambda regimes facilitate inflation expectations having a greater influence on yield determination.
\[ \eta_{S_t} = \eta_0 (1 - S_t) + \eta_1 S_t, \quad S_t = 0,1. \] (21)

Although this model allows for switching in both the factor loadings and factor volatilities, it is limited by forcing the loadings and volatilities to switch simultaneously.\(^{12}\) But this limitation allows macroeconomic fundamentals to explain regimes of high or low volatility purely through the construction of the model. For example, given a low volatility-lambda regime, a low lambda value dictates a slower decay of the slope and curvature loading factors. This means those factors proxying for monetary policy and real economic activity has greater influence on the various maturities of the term structure. Kim et al. (2007) develop a framework for a more generalized Markov-switching model that allows timing of switches to differ across model components that experience switching.

3.3 Data

The end of the month zero-coupon government yields data is constructed using piecewise cubic polynomials to model forward rates. Polynomial coefficients are restricted so that the polynomial function is of differentiable class \(C^1\) at all maturities. Segments are connected knot points. The yields were constructed by Anderson and Sleath (1999) and supplied by Jonathan Wright (2011). The maturities under investigation are the 3, 6, 9, 12, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120-month over the period Jan. 1985 through Dec. 2006 when analyzing the link between NS factors and macro-factors with the MFA-DNS model and Jan. 1979 through Dec. 2006 for the regime switching analysis with the DNS-MSL, DNS-MSV, and DNS-MSLV models. In Table 3.1, I present descriptive statistics for the various maturity yields.

The increasing pattern of the yield means as we go to longer maturities indicates an upward sloping yield curve generally for the UK term structure.

\(^{12}\) Kim et al. (2007) develop a framework for a more generalized Markov-switching model that allows timing of switches to differ across model components that experience switching.
I utilize a monetary policy-related interest rate variable, industrial production, and inflation expectations as unspanned factors to create a macro-factor augmented state-space framework. The policy interest rate is introduced into the framework as the annual percent change in the less than 24 hour interbank rate for the United Kingdom. Industrial production data is also transformed into annual growth rates for the purposes of this analysis. Inflation expectations come from the Consumer Opinion Surveys of the future tendency of inflation compiled by the European Commission and National Indicators for the United Kingdom. This data enters into the state-space framework as net percent. Data on the three macro-factors come from the FRED database for the period Jan. 1985 through Dec. 2006. I present the descriptive statistics for these macro-factors at the bottom of Table 3.1. Figure 3.1 gives a visual for the time-series dynamics of the 3mo, 24mo, and 120mo maturities along with the macro-factors used.

3.4 Empirical Results

In this section, I present the estimation results of a yields-macro model and three Markov-switching yields models to better understand the macroeconomic causes of regime changes in the UK term structure. I utilize maximum likelihood estimation via the Kalman filter to obtain smoothed parameter estimates and smoothed probabilities for the yields-macro model. Markov-switching models are estimated using the Kim (1994) algorithm which allows inference of the hidden Markov component in all switching models. Details of the estimation procedure can be found in Kim and Nelson (1999).

3.4.1 Yields-Macro Model Results

As discussed in sub-section 2.2, I choose an identification strategy for estimation of the MFA-DNS model that allows for off diagonal elements in the FAVAR(1) coefficient matrix of
Eq. (9) and restricts the factor variance-covariance to be diagonal. Parameter estimates of this model are given in Table 3.2. Many of the off-diagonal elements are insignificant for the coefficient matrix, but the significant coefficients give insight into the interaction of the yield curve and macroeconomic fundamentals. The growth rate of the monetary policy-rate has significant dependence only on the curvature factor while the growth rate of industrial production has significant dependence on all three yield curve factors within the state-space framework. The negative sign associated with slope factors (-0.13) points to a relationship between economic activity and the slope factor through counter-cyclical monetary policy. Wu (2002) and Estrella et al. (2003) have documented this result using US yields and macro-factor data. The relatively large coefficient (0.16) of the level factor for inflation expectations gives support to the connection inflation expectations has with the level factor. Estimates of the macro-factor unconditional means are close to values calculated in the descriptive statistics of Table 3.1, lending support that the maximum likelihood value is indeed the maximum or very close to the maximum for this highly non-linear model.

Smoothed factor estimates are calculated using the parameter estimates presented in Table 3.2. Figures 3.2-3.4 present the time series of the smoothed level, slope, and curvature factors, respectively. Each figure also contains plots of corresponding observed yield curve factors and macro-factors as a means of comparison. Observed, or empirical, factors are calculated in the following manner:

\[
\text{Level} = \frac{y(3) + y(24) + y(120)}{3}
\]

\[
\text{Slope} = y(3) - y(120)
\]

\[
\text{Curvature} = 2\cdot y(24) - (y(120) - y(3))
\]
Figure 3.1 shows the smoothed level and empirical level match each relatively closely throughout the sample period. Inflation expectations are more volatile than both level time-series but follow the same general downward trend. Smoothed level and inflation expectations have a correlation of 0.78, as presented in Table 3.3. The smoothed slope factor is shown in Figure 3.2 along with its empirical counterpart and the policy interest rate. With how well the plots of the smoothed slope factor and the policy rate resemble one another, it can be concluded that slope and monetary policy are related for the UK. They have a correlation of 0.51 which supports this visual finding.

The plot for the curvature factor and industrial production are shown in Figure 3.3. Sometime after 1990, the two plots appear to display similar trajectories despite the high volatility of industrial production. The correlation for these two is 0.17 which is no indication that the curvature factor is related to economic activity, but in the next sections I find evidence that such a relationship does indeed exist.

3.4.2 Impulse Response Analysis: 1985.1-2006.12

This section investigates the dynamic relationship between the yield curve factors and macroeconomic fundamentals over the entire sample period (1985.1-2006.12) through impulse response analysis. I discuss four groups of responses: macro responses to macro shocks, macro responses to yield shocks, yield curve responses to macro shocks, and yield curve responses to yield shocks. All impulse responses are presented with 95% confidence bands.

The macro-factor responses to macro shocks in this paper follow suit with the findings of DRA and Rudebusch and Svensson (1999). Figure 3.5 shows the high level of persistence for the macro-factors given macro shocks. An increase in industrial production gives rise to a persistent increase in the policy rate which counters the effects of a possibly overheating
economy by raising the cost of capital. Industrial production slightly increases in response to an increase to the policy rate, is counterintuitive to economic theory. This may occur as a result of an unclean identification of the policy shock which can give the perception that monetary policy causes output to rise. The policy rate also rises with inflation expectations which follow from a Taylor rule-type reaction function. Inflation expectations increase with an increase to capacity utilization representative of an aggregate supply response and expectations diminish over time with increases to the policy rate.

The macroeconomic fundamentals response to yield curve shocks offers interesting insight into the interaction of the UK macroeconomy with the term structure. An increase in the level factor increases both inflation expectations and industrial production but depresses the policy rate for a period then raises it. An increase to slope decreases all three macro-factors. If slope is proxying for monetary policy then decreases in inflation and economic activity follows from economic theory. In contrast, shocks to the curvature factor caused increases in all three macro-factors with persistent increases in inflation expectations and industrial production. This would follow if curvature proxies for economic activity where an increase in inflation expectations is indicative of an aggregate supply response and an increase in the policy rate is representative of a central bank suppressing increased economic activity to counter future inflation.

Yield curve responses to macro shocks complete the duality of the interaction between the macroeconomy and the term structure. Similar to DRA in their impulse response analysis for the US, this paper finds the curvature factor shows negligible response to macro-factors. The slope factor experiences persistent increases from positive shocks to inflation expectations and the policy rate. The response of the slope factor to these two macro-factors is in accordance with
a central bank making the yield curve less positively sloped by raising the short end of the term structure to mitigate the economic effects of inflation surprises. The level factor is affected by all three macro-factors. Positive shocks to industrial production and inflation expectations cause persistent increases in the level factor. This is indicative of the problem UK monetary policy had prior to 1992 with anchoring inflation expectations. Surprises in inflation cause expectation of high future inflation, which raises the level of the term structure. In contrast, the level factor exhibits a persistent decrease to positive shocks to the policy rate. As DRA explains, a surprise increase in the policy rate could potentially have two opposite effects on inflation expectations. A surprise tightening could be an intimation of a lower inflation target which would depress the term structure level, given the bank has a large degree of credibility and transparency. On the other hand, an unexpected tightening could indicate inflationary pressure worries which would raise inflation expectations. The 1985.1-2006.12 sample used is dominated by the former effect which would have taken effect post-1992 with inflation-targeting and increased transparency of monetary policy decisions.

Lastly, an analysis of yield curve responses to yield curve shocks shows a great degree of persistence along the diagonal. Increases in the level factor raises both the slope and curvature factors, although the curvature factor increases slightly and both responses are negative. Positive shocks to slope have a negligible negative effect on curvature but a persistent positive effect on level. This is in accordance with DRA’s second explanation from above which would correspond to the pre-1992 period. Positive shocks to curvature resulted in persistent increases in both level and slope. This result potentially links curvature with real economic activity. Surprise increases in economic activity would lead to increases in inflation expectations.
and inflationary pressures which would cause a central bank to make the yield curve slope less positive to curb such pressures.

3.4.3 Impulse Response Analysis: 1985.1-1992.10

In September 1992, the UK exited the Exchange Rate Mechanism of the European Monetary System. Three weeks later on October 8, 1992 the Chancellor of the Exchequer, Norman Lamont, established an inflation target of 1-4% for the annual retail price index (RPI). I impose the October 1992 break date in the following impulse response analyses to explore how this marked change in monetary policy affected the interaction between the bond market and macro-economy. I limit the discussions of the next two sub-sections to the impulse responses of macro responses to macro shocks, macro responses to yield curve shocks, and yield curve responses to macro shocks.

This section investigates the interaction of the UK term structure and macroeconomy for the period 1985.1-1992.10. During this period UK monetary policy targeted monetary aggregates such M0 (narrow money) as a way of coralling inflation but found monetary aggregates to be poor nominal anchors for inflation. Exchange-rate-based nominal anchors were then utilized from 1987 until those proved poor anchors as well and the UK left the ERM.

Figure 3.6 displays the impulse responses under investigation. The negligible effect on industrial production and inflation expectations that a shock to the level factor caused supports the narrative of poor nominal anchors for inflation that plagued the UK during this period. A shock to inflation expectations increased the overall level of yield curves which is another signal that inflation expectations are not firmly anchored. Increases to the curvature factor caused a significant increase in industrial production but, increases in industrial production had a negligible effect on yield curve factors. An increase in the policy rate raised inflation
expectations and industrial production. BMS finds a similar result in their impulse response analysis. An increase in the policy rate also increases slope, and an increase in slope leads to less persistent increase in the policy rate. Overall, aside from the slope factor, there appears to be minimal yields to macroeconomic fundamentals interaction and more macroeconomic fundamentals to yields interaction for the period where UK monetary policy targeted monetary aggregates and exchange rates as nominal anchors for inflation.


Inflation-targeting appears to correct the macroeconomic inefficiencies of the pre-1992 period. Impulse responses are presented in Figure 3.7. Increases in the level factor give rise to persistent increases in both inflation expectations and industrial production and has negligible effect on the policy rate. A shock to inflation expectations increases the level factor which supports the link between the two. The increase in inflation expectations also depresses slope which is consistent with a less steep yield curve in response to a potentially “overheating” economy. Increases to industrial production leads to a highly significant and persistent increase in curvature. Similarly, increases in the curvature factor cause a significant and persistent response in industrial production. This finding casts light on the elusive nature of the curvature factor. During the inflation-targeting era of the UK, real economic activity is related to the curvature factor. The impulse response relationship between shocks to slope and shocks to the policy rate are consistent across monetary regimes where a shock to either causes an increase in the other. An increase in the policy rate also depresses inflation expectations and industrial production which is more consistent with economic theory than what occurred in the pre-1992

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13 This counter-intuitive result regarding the policy rate and industrial production was addressed previously in subsection 4.2.
period. Aside from the slope factor, there appears to be evidence of a bi-directional yields-macro interaction for the post-1992 era.

3.4.5 Markov-switching Models Results

Equipped with concrete linkages between yield curve factors and macroeconomic fundamentals, I estimate Markov-switching dynamic Nelson-Siegel models to model parameter instability in the UK term structure and relate observed regimes to macroeconomic fundamentals. I first estimate the term structure using the DNS-MSL model which directly relates regimes to macroeconomic fundamentals through the application of a hidden-Markov switching component to the factor loading parameter, $\lambda$. Parameter estimates are presented in Table 3.4. There is significant switching in $\lambda$ evidenced by the small standard errors for the estimated values – 0.0203 (0.001) and 0.101 (0.003). Although factor plots for this model resemble those of the MFA-DNS model, this model suffers from mis-specification as evidenced by the insignificant unconditional means for the yield curve factors. Factor plots are presented in Figure 3.8. Despite this shortcoming, the DNS-MSL gives insight into regimes where yields determination was heavily influenced by inflation expectations as can be observed in the smoothed probability plot of Figure 3.11.\[14\]

The second model estimated to capture parameter instability is the DNS-MSV model. Although it does not have the capabilities of relating regimes to macroeconomic fundamentals, it is still worthwhile to estimate because it will allow us to see if the Markov-switching framework has the ability to capture the UK Great Moderation. Parameter estimates are presented in Table 3.4 and factor plots are presented in Figure 3.9. The likelihood value of this model is substantially larger than that of the DNS-MSL model. Also, the AIC and BIC values are smaller

\[14\] High lambda regimes correspond to regimes where the loading factor parameter decays quickly diminishing the effects of slope and curvature, leaving only the level factor to determine yields.
for this model than the previous model. Figure 3.12 shows the smoothed probability plot of the regime changes for the term structure. There is evidence of a substantial decrease in term structure volatility post-1992 with more frequent and persistent low-volatility regimes appearing.

The third and final switching model estimated is more general than the previous two models in that it allows for the factor loadings and factor volatilities to switch simultaneously. The parameter estimates of the DNS-MSLV model show significant switching in the factor loading parameter and factor volatilities. In both states, the curvature factor exhibits the highest volatility. The maximum likelihood value is the highest of all estimated switching models. AIC and BIC values are the smallest for the DNS-MSLV giving credence to the evidence that this model most accurately models the UK term structure of the switching models proposed in this paper.

Smoothed factor plots are presented in Figure 3.10 and the smoothed probability plot is presented in Figure 3.13. It is a bit clearer that a great moderation took place post-1992 with the smoothed probability plot of this model than with the DNS-MSV model. And since the DNS-MSLV model also restricts the factor loadings to switch at the same time as the factor volatilities, a low-volatility regime corresponds to a low-lambda regime and vice versa. Therefore, periods of low volatility correspond to periods in which economic activity and monetary policy have a greater influence on the bond’s market. This would support the argument of those that find good policy being the cause of more stable macroeconomic conditions in the UK during the inflation-targeting regime. High volatility regimes in Figure 3.13 are explained by periods when inflation expectations dominate yield pricing—as the model dictates through the factor loading parameter— and post-1992 those periods are more infrequent given the inflation-targeting policy instituted.
3.5 Conclusion

This paper investigates the UK yields-macro interaction in response to changes in monetary policy. The October 1992 monetary policy change to inflation-targeting from exchange-rate and monetary aggregate anchoring for inflation is employed as a reference to compare the yields-macro interaction across monetary regimes. Using a macro-factor augmented dynamic Nelson-Siegel yield curve model this paper establishes concrete links between each yield curve factor and macroeconomic fundamentals for the UK. I perform correlation and impulse response analyses for the entire sample period spanning 1985.1-2006.12, the 1985.1-1992.10 sub-sample and 1992.11-2006.12 sub-sample.

The level factor appears to be directly related to inflation expectations across monetary regimes. The slope factor is related to monetary policy across regimes. The elusive link between curvature and macroeconomic fundamentals becomes less obfuscate during the inflation-targeting regime where it is related with real economic activity. The correlation between the curvature factor and industrial production achieves its highest value post-1992 and impulse responses point to a highly significant bi-directional interaction between the two during this period.

With a clear match between each of the yield curve factors and macroeconomic fundamentals used, I utilize Markov-switching dynamic Nelson-Siegel yield curve models to explain the parameter instability in the UK term structure for the period 1979.1-2006.12. I find that evidence of a great moderation in the term structure volatility after 1992 as evidenced by an increased frequency of low volatility regimes in smoothed probability plots. By construction of the estimated model, these low volatility regimes correspond to regimes where real economic activity and monetary policy have greater influence on the bonds market. On the other hand,
once again by construction of the model, regimes of high volatility coincide with regimes where inflation expectations have a greater influence on yield determination.
3.6 References


Table 3.1
Descriptive Statistics

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Macro-Factors

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Estimated Q matrix

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Maximum Likelihood Value: 7914.3
Loading Parameter ($\lambda$): 0.060
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Max. Likelihood Value: 9698.52, 11763.5, 12130.7

Number of free parameters ($k$): 30, 32, 33

AIC: -19337.0, -23463.0, -24195.4

BIC: -19137.5, -22250.2, -23975.9

Standard errors are in parentheses.

Implied maturities measured in months are in brackets.

We calculate the information criteria according to the formulas $AIC = -2 \cdot \ell(\theta)_{\text{max}} + 2 \cdot k$ and $BIC = -2 \cdot \ell(\theta)_{\text{max}} + k \cdot \ln(NT)$, where $N = 17$ maturities and $T = 336$ months.
Figure 3.1: Maturities and Macro-Factor Time-Series Plots
Figure 3.2: Smoothed Level Factor and Inflation Expectations Plot
Figure 3.3: Smoothed Slope Factor and Policy Rate Plot
Figure 3.4: Smoothed Curvature Factor and Industrial Production Plot
Figure 3.5: Impulse Responses 1985.01 – 2006.12
Figure 3.6: Impulse Responses 1985.01 – 1992.10
Figure 3.7: Impulse Responses 1992.11 – 2006.12
Figure 3.8: DNS-MSL Smoothed Factors
Figure 3.9: DNS-MSV Smoothed Factors
Figure 3.10: DNS-MSLV Smoothed Factors
Figure 3.11: DNS-MSL Smoothed Probability Plot
Figure 3.11: DNS-MSV Smoothed Probability Plot
Figure 3.11: DNS-MSLV Smoothed Probability Plot